Instability of Isothermal Stellar Wind Bowshocks

John M. Blondin [1] and Joel F. Koerwer [2]

Department of Physics, North Carolina State University, Raleigh NC

Abstract

We present hydrodynamical simulations illustrating the instability of stellar wind bowshocks in the limit of an isothermal equation of state. In this limit, the bowshock is characterized by a thin dense shell bounded on both sides by shocks. In a time-averaged sense the shape of this bowshock shell roughly matches the steady state solution of Wilkins (1997), although the apex of the bowshock can deviate in or out by a factor of two or more. The shape of the bowshock is distorted by large amplitude kinks with a characteristic wavelength of order the standoff distance from the star. The instability is driven by a strong shear flow within the shock-bounded shell, suggesting an origin related to the nonlinear thin-shell instability. This instability occurs when both the forward bowshock and the reverse wind shock are effectively isothermal and the star is moving through the interstellar medium with a Mach number greater than a few. This work therefore suggests that ragged, clumpy bowshocks should be expected to surround stars with a slow, dense wind (which leads to rapid cooling behind the reverse wind shock), whose velocity with respect to the surrounding interstellar medium is of order $60\,\mathrm{km\,s^{-1}}$ (leading both to rapid cooling behind the forward bowshock and sufficiently high Mach numbers to drive the instability).

Key words: hydrodynamics, instabilities, shock waves, circumstellar matter
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1 Introduction

The presence of a strong stellar wind is common in early type stars, where the tremendous luminosity carries sufficient momentum to drive a mass flux from

[1] Email: john_blondin@ncsu.edu
[2] Email: jfkoerwe@unity.ncsu.edu

the star of 10^{-7} to 10^{-5} solar masses per year. Over time this stellar wind will form a low density bubble surrounded by a shell of swept up interstellar gas. The structure and evolution of these stellar wind bubbles evolving within a uniform medium is described in detail by Weaver et al. (1977). For typical wind parameters and stellar lifetimes, these stellar wind bubbles can reach radii of several tens of parsecs.

This picture will be modified if the star is moving with respect to the local ISM. If this relative velocity is larger than the sound speed in the ISM, the spherical bubble will instead be a bowshock pointing in the direction of stellar motion. There are many examples of stellar wind bowshocks associated with runaway OB stars (Van Buren & McCray 1988). These systems possess both the strong stellar wind and the high space velocity needed for a prominent bowshock. These bowshocks are typically seen via the infrared emission of heated dust in the bowshock shell. A particularly striking example of a stellar wind bowshock is seen in Hα around the high-mass X-ray binary system Vela X-1 (Kaper et al. 1997).

The characteristic size of a bowshock is determined by the balance between the ram pressure of the stellar wind and the ram pressure of the on-rushing ambient medium into which the bowshock is moving. Since the ram pressure in the stellar wind drops as r^{-2} for a constant velocity wind, there will be some radius at which the ram pressure in the wind is equal to that in the ambient medium. The radius at which this happens is referred to as the standoff distance, and is given by (Baranov et al. 1971)

$$R_{so} = \left(\frac{\dot{M}_w v_w}{4\pi \rho_o v_*^2} \right)^{1/2} .$$

Here $\dot{M}_w$ and v_w are the mass loss rate and terminal velocity of the stellar wind, v_* is the velocity of the star with respect to the local ISM, and ρ_o is the ambient density. In the case of Vela X-1, Kaper et al. (1997) measure a standoff distance of $R_{so} \approx 0.5$ pc. Together with their estimated wind parameters of $\dot{M}_w \approx 10^{-6}\,\mathrm{M_\odot yr^{-1}}$, $v_w \approx 1100\,\mathrm{km\,s^{-1}}$, and a velocity of the star through the ISM of 90 km $\mathrm{s^{-1}}$, they derive a reasonable value for the ambient density of $\rho_o \approx 2.6 \times 10^{-24}\mathrm{g\,cm^{-3}}$.

Because the stellar wind itself is typically highly supersonic, the wind will terminate in a shock on the back side of the bowshock. The bowshock is thus a layer of gas bounded on both sides by shock fronts: the forward shock on the leading edge propagating into the ISM, and the termination shock on the back edge decelerating the stellar wind. Within this layer is a contact discontinuity separating the shocked ISM from the shocked stellar wind. Both of these shocked gasses will radiate through a variety of processes. If the gas density is sufficiently large that these radiative processes can remove thermal energy on a timescale short compared with the flow time, this layer of gas will collapse to a thin, dense shell. The width of this shell will thus be determined by the efficiency of radiative cooling; the faster the cooling, the thinner and denser the shell.

It is not unusual for the forward shock to be rapidly cooling. For the case of stellar wind bowshocks, we shall define rapid cooling as the limit in which the postshock cooling length, L_c, is much less than the standoff distance, R_{so}. A rough estimate of the cooling length is given by the postshock velocity times the thermal energy density divided by the local cooling rate of the gas. In terms of preshock quantities this gives us

$$L_c \approx \frac{3}{256}\frac{\bar{m}^2 v_*^3}{\rho_o \Lambda},$$

where $\bar{m}$ is the average mass per particle and Λ is the volume cooling rate in erg cm^3. For a typical stellar velocity of $\sim 50\,\mathrm{km\,s^{-1}}$ and a corresponding cooling rate of order $10^{-21}\,\mathrm{erg\,cm^3}$, an ambient density of $n = 1\mathrm{cm}^{-3}$ leads to a cooling length of only 1.5×10^{15} cm, far smaller than a typical standoff distance. However, for typical OB wind velocities (thousands of km s^{-1}) the postshock cooling length of the termination shock is much greater than R_{so}, making the shock effectively adiabatic. If, however, the cooling is enhanced by molecular emission, or the wind velocity is low, and/or the star is moving through relatively dense ISM such that R_{so} is small, then the termination shock may also be rapidly cooling. A likely case for a cooling termination shock is a bowshock around red supergiant stars. Winds from these cool supergiants are both slow and dense, both of which enhance the effectiveness of radiative cooling. An example of such a star is Betelgeuse, around which Noriega-Crespo et al. (1997) have imaged what appears to be a bowshock or asymmetric shell.

If both the bowshock and the wind termination shock are rapidly cooling, the hydrodynamics of the bowshock can be treated in the limit of an isothermal gas. In this limit the shell density is M^2 times the preshock density, where M is the Mach number of the shock. For large Mach number the shell is very dense and correspondingly thin. In the limit of an infinitely thin shell (i.e., for infinite Mach number) an analytic expression for the shape of the bowshock shell can be derived through conservation of mass and momentum (Wilkin 1996). The radius of the bowshock as a function of angle from the apex is given by

$$R = R_{so} \csc\theta \sqrt{3(1 - \theta \cot\theta)}.$$

The geometry of the problem and this equilibrium solution are shown in Figure 1.

In the limit of an infinitely thin bowshock, Dgani et al. (1996) suggest that the bowshock is dynamically unstable to the transverse acceleration instability

(TAI). The analysis leading to the description of the TAI was done in the zero-thickness shell limit, such that the instability depends only on the integrated properties of the shell and not on the structure within it. The source of the TAI is a lack of balance between the ram pressures on the two sides of the shell when it is displaced from its equilibrium. The result is that a bent section of the shell will be pushed away from the face of that section that now intercepts more momentum than in the equilibrium position.

Dgani et al. (1996) note that the nonlinear thin shell instability (NTSI) may also affect the evolution of an isothermal bowshock. It has been shown by analytic analysis (Vishniac 1994) and confirmed by numerical simulation (Blondin & Marks 1997) that an isothermal shock-bounded slab with a displacement larger than the width of the slab will be susceptible to the NTSI. The NTSI is driven by the shear flow generated by a bent slab. The bounding shocks divert the interior flow toward the points of maximum displacement of the slab. The enhanced momentum flux reaching these extrema push these parts of the slab to even larger displacement. The result is that a nonlinear "wiggle" in a thin, shock-bounded shell will grow to large amplitudes.

Dgani et al. (1996) compare the growth rates for the NTSI and the TAI and argue that at best, the NTSI provides an initial seed for the TAI. They quote a growth rate for the TAI of $\sqrt{kl}v_w/R_{s0}$ for $l \leq R_{so}$. Here l is the distance along the bowshock as measured from the stagnation point ($\theta = 0$), as illustrated in Figure 1, and $k = 2\pi/\lambda$ is the wavenumber of the unstable mode. For the NTSI they quote a growth rate of order $c_s k$, where c_s is the sound speed in the gas. Based on their numerical simulations, Blondin & Marks (1997) find a growth rate that differs from this by only $0.4/(kL)^{0.3}$, where L is the width of the slab. Since the most unstable wavelengths correspond to kL of order unity, a rate of $c_s k$ is a good approximation. Comparing these growth rates one finds that the TAI grows faster than the NTSI if

$$\frac{l}{R_{so}} > \frac{2\pi}{M^2}\frac{R_{so}}{\lambda}.$$

Since the relevant wavelengths must be less than or of order R_{so}, and a typical Mach number may be of order 10, this relation implies that the TAI grows faster than the NTSI only out in the bowshock wings ($l > R_{so}$). For larger Mach numbers the region of dominance by the NTSI shrinks, but it is always the more important instability at the apex of the bowshock. However, out in the wings of the bowshock the instability is being advected away at least as fast as it grows, so there is little chance for significant growth. At the apex of the bowshock, where the NTSI clearly dominates, the advection timescale is much longer than the growth timescale, and the NTSI has sufficient time to grow to large amplitudes. By this reasoning, the NTSI is responsible for rapid distortions of the bowshock near the apex, while the TAI may amplify these distortions as they advect out the bowshock wings.

Predicting the relevant role of the NTSI and the TAI is not just a matter of comparing growth rates, for the assumption of an infinitely thin shell as used in the analysis of the TAI is inconsistent with the presence of interior shear flow responsible for the NTSI. The TAI considers only the net gain of momentum from the two sides of the shell, while the NTSI considers the specific postshock flow of momentum from each side of the shell. In fact the NTSI follows directly from the nature of the interior flow for a bent slab, with positive momentum from one side flowing in one direction within the slab, and negative momentum from the other side flowing in the opposite direction. The presence of such a shear flow violates the thin shell approximation assumed for the TAI. On the other hand, the analysis and simulations of the NTSI have only been done for planar flow, and do not account for the diverging flow of the stellar wind.

While it has been suggested that isothermal bowshocks are unstable, it is not clear which is the relevant instability, nor is the final outcome of the instability known. We propose to address these questions using hydrodynamic simulations of an idealized stellar wind bowshock.

2 Numerical Model of an Idealized Bowshock

In this study, an isothermal variant of the VH-1 hydrodynamics code was implemented in two and three dimensions in the rest frame of the star. The 2D simulations were run on a spherical coordinate system assuming axisymmetry around the direction of motion of the star. The spherically diverging wind was modeled by setting the radial velocity to the appropriate magnitude and the tangential component to zero at the inner radial boundary of the grid. The incoming planar wind was created by fixing the flow variables at the outer radial boundary for $\theta < \pi/2$, and allowing outflow off the grid for $\theta > \pi/2$. The grid was set up so that the ram-pressure balance of the stellar wind and the on-rushing ISM balanced in the center of the grid. The parameters of the problem were normalized such that the sound speed, $c_s = 1$ everywhere and the density of the ISM, $\rho_o = 1$. We initially assumed that the wind velocity and stellar velocity were equal. The problem then reduces to a single parameter, the Mach number of the winds. Simulations were run for each wind (stellar and ISM) of Mach 2.5, 5 and 10 at resolutions of both 120^2 and 240^2 zones. Additional runs were completed for Mach 5 and 10 with a resolution of 480^2 zones. We then allowed for different values of the stellar wind and space velocities, introducing a second parameter, $\beta \equiv v_w/v_*$. Finally, we ran a three dimensional simulation on a Cartesian grid to check the effects of restricting the geometry to axisymmetry.

3 Results of Numerical Simulations

3.1 Instability of Bowshock

The evolution of a Mach 10 stellar wind bowshock is shown in Figure 2 and the accompanying animation. The shape of the bowshock is roughly coincident with the analytical solution of Wilkin (1996) for an infinite Mach number steady-state bowshock, but the bowshock is clearly neither smooth nor steady.

This simulation is initialized with a thin shell geometry given by the Wilkins solution, and although the overall shape of the bowshock stays close to this solution at early times, small wiggles distort the shell almost immediately after the evolution begins. These initial wiggles have a wavelength a few times less than R_{so}, and show up along the entire length of the bowshock. These wiggles grow fastest at positions near, but slightly off the axis of symmetry. The weaker growth very close to the axis is likely a result of the forced axisymmetry, while the weaker growth out in the bowshock wings can be attributed to the lower Mach number of the oblique shocks, particularly on the ISM side of the bowshock. Out in the bowshock wings this slower growth rate is comparable to, or even slower than, the advection rate out along the wings. The small wiggles do not grow much before they are advected out the bowshock wings and off the numerical grid. In contrast, the wiggles near the point of maximum growth appear to grow almost in place, and are only gradually advected out into the wings. This early growth, particularly in the region of highest growth rate, is clearly reminiscent of the NTSI (cf. Blondin & Marks 1997).

After a few flow times (R_{so}/v_w) larger wavelength wiggles with $\lambda \sim R_{so}$ begin to dominate the shape of the shell. These large wavelength modes originate at or near the apex and grow to an amplitude of roughly R_{so}. On approximately the same timescale that these bends in the shell grow, they are advected into the wings of the bowshock. Thus, the instability remains, in a sense, bounded, with the shell never completely disrupting.

At late times the evolution appears to be dominated by the motion of the apex oscillating in and out with respect to the equilibrium position. In this high-Mach number simulation this oscillation becomes so violent that the bowshock flows off the grid at $4R_{so}$, ending the simulation. All of our Mach 10 simulations ended this way, but all lower-Mach number simulations remained on the grid for as long as we evolved them. Although we expect the apex of the bowshock to be the site of the highest instability growth, this dominance of the apex and its dramatic run off the grid is again likely an artifact of axisymmetry.

The evolution of the shell and the internal flow suggest this instability is related to the NTSI. Once a significant bend appears in the shell, the amplitude of the bend grows as the extrema are driven away from each other, shearing the shell apart. The internal flow structure, shown in Figure 3, shows the internal shear characteristic of the NTSI. The postshock flow diverts the gas (and its momentum) towards the extrema of the bend. It is this directed advection of momentum that increases the momentum of the extrema and drives them farther apart.

There is also evidence that at least the bending mode of the TAI is operating in the wings of the bowshock. In Figure 4 the geometry of the winds and shell are shown for a region of the bowshock out in the wing. At this particular moment it is clear that the stellar wind is imparting more momentum flux into the shell than the ISM wind since the stellar wind is impacting in a nearly perpendicular shock, while the ISM wind is striking the shell very obliquely. The net result is that this section of the shell is accelerated out away from the star, further distorting the shape of the bowshock. This lateral acceleration of vertical segments is clearly seen in the animation accompanying Figure 2. When a segment of the bowshock wing is both thin and more vertical than the equilibrium solution, it can be seen to move laterally outward, increasing the width of the bowshock. This effect can be quite dramatic, increasing the overall width of the bowshock by a factor of 2 and producing an unusually flat-faced bowshock. Note however that this TAI is only seen to operate in the nonlinear limit, i.e., the shell is already strongly perturbed from equilibrium by the action of the NTSI. There is no evidence that the TAI could significantly distort the bowshock from small linear perturbations in the absence of the NTSI.

Before discussing the dependence of this instability on physical parameters, let us briefly address the question of numerical resolution. One of the primary concerns when analyzing numerical simulations is the accuracy of the results. This is particularly true with fluid instabilities that involve a wide range of length scales, for the fluid dynamics on scales smaller than a grid zone are completely ignored. To address this concern we ran simulations for the Mach 5 model at 120^2 zones, 240^2 zones, and 480^2 zones. An image from each bowshock simulation is shown in Figure 5, corresponding to an arbitrary time $t = 6$. Ideally, as resolution increases, the simulation should converge on an 'exact' solution, although we note that the instability here is seeded only by numerical 'noise', which is itself resolution-dependent. Although these three simulations show some qualitative similarity, there is clearly no convergence up to 480^2 zones. However, the amplitude of the shell displacements and the dominant wavelength show little dependence on resolution.

3.2 Dependence on Mach Number

Because the NTSI requires a perturbation at least on the order of the width of the slab, one would expect thinner slabs to be more susceptible to the NTSI. Furthermore, Vishniac's assumption of small bending angles translates into a requirement that $\lambda \gtrsim 2\pi L$. Since we expect the relevant wavelengths to be on the order of R_{so} or less, this requires that $L \lesssim R_{so}/2\pi$. For an isothermal shock, the shell thickness decreases and the density increases with higher Mach number. From the shock jump conditions, the internal density of a planar shocked slab is greater than the density of the incoming flow by a factor of the Mach number squared. Using this result and balancing the mass flux into the shell with the mass flux flowing out through the shell in the vicinity of the apex, one finds that the shell thickness at the apex is $L \approx R_{so}/M^2$. Our simulations with different Mach numbers confirm this general trend, although for low Mach numbers this expression is not quantitatively accurate (the shell is thicker), and for high Mach numbers the shell is not stable long enough to accurately estimate the steady-state shell thickness.

These arguments suggest that the Mach number must be at least a few in order for the bowshock to be unstable, and that the instability becomes more effective as the Mach number increases. The simulations shown in Figure 6 confirm this expectation. The lowest Mach number simulation has a relatively stable shell with $L \approx 0.7R_{so}$ at the apex. There are some surface waves along the termination shock that generate small vorticies from kinks in the shock surface, but these vorticies simply advect out the bowshock wings with no significant effect on the overall shape of the bowshock.

At higher Mach number the bowshock shell is indeed thinner and unstable. Although one can see shorter wavelength perturbations in the Mach 10 simulation compared to the Mach 5 (particularly near the apex), the dominant wavelength appears to remain of order the standoff distance from the star. The simulations shown in Figure 6 also show that the amplitude of the bends is noticeably larger in the Mach 10 simulation. In fact, in every simulation of a Mach 10 bowshock the apex eventually propagated off the edge of the grid, while in all of the Mach 5 simulations the apex remained within $4R_{so}$.

3.3 Dependence on Speed Ratio

We ran several simulations for $\beta \neq 1$ to investigate the relative importance of v_w and v_*. Although the strength of the instability is not easy to quantify, one can compare the relative instability of two simulations with reasonable accuracy. Thus, in Table 1 we list the different simulations ranked according

Table 1
Dependence of Instability on Speed Ratio

v_*	v_w	β	rank
2	4	2.00	most stable
2	8	4.00	.
3	6	2.00	.
3	9	3.00	.
4	2	0.50	.
4	8	2.00	.
5	5	1.00	.
6	3	0.50	.
8	2	0.25	.
8	4	0.50	.
9	3	0.33	most unstable

to the strength of the instability. From this ranking it is clear that the stability depends primarily on the Mach number of the star through the ISM, and depends only weakly on the Mach number of the stellar wind.

This result is in contrast to the dependence on β predicted for the TAI. Dgani et al. (1996) find that bowshocks are more unstable to the TAI for stars with slow winds, i.e., $\beta < 1$. While a glance at Table 1 suggests that $\beta < 1$ is indeed more unstable, this is due not to the value of β but to the value of v_*. Consider, for example, the two simulations with $v_* = 4$. The case with a slow wind is actually more stable than the case for a fast wind, contrary to the prediction for the TAI.

To illustrate this dependence on the speed ratio, β, we show in Figure 7 a representative time slice from two different simulations. In the first example we show a simulation with $v_* = 2$ and $v_w = 8$ that appears relatively stable despite the Mach number of the stellar wind being very high. In the second case we switch the velocities such that $v_* = 8$ and $v_w = 2$ and find that the bowshock exhibits a very strong instability. This comparison illustrates that it is not just a high Mach number that is needed for instability, but specifically a high Mach number of the star through the ISM.

3.4 Three Dimensional Bowshock

We must stress that the above results are derived for strictly axisymmetric bowshocks, i.e., the instability is operating as concentric rings. While we expect our general conclusions to hold at least qualitatively for more realistic bowshocks, the only sure way to confirm our expectations is to run a three dimensional simulation. To this end we ran a model with $\beta = 1$ and Mach = 5 on a three dimensional Cartesian grid of 160 by 240 by 240 zones. The grid spanned a physical size of $-1 < x < +3$, $-3 < y < +3$, and $-3 < z < +3$ in units of the nominal standoff distance, with the planar flow of the ISM approaching from the direction of positive x. This limited spatial domain does not allow us to follow the flow far into the bowshock wings, but is sufficient to ensure that the flow through the wings, out the boundary, is supersonic. A steady, isotropic stellar wind was created by resetting an inner region of 48 zones in radius at each timestep.

Isothermal bowshocks are at least as unstable in three dimensions as in two. Figure 8 shows a snapshot from this simulation along with that of an axisymmetric 2D simulation with comparable numerical resolution. Again, the dominant wavelength in both cases is of order R_{so}, although it is slightly smaller in the 3D simulation. However, there is substantially more small-scale structure and more complicated internal flow in 3D as compared to 2D. The peak densities in the bowshock are comparable in the two simulations, with typical values of order 10 and ranging up to ~ 30 in the densest knots. This is comparable to the nominal peak density in the equilibrium bowshock of $M^2 = 25$. However, these knots are substantially more compact and numerous in the full three-dimensional simulation. The three-dimensional nature of the instability is best seen in the animation in Figure 9 showing the evolution of the outer shock surface. Here one can more clearly see the presence of many smaller wavelength perturbations, as well as the dominance of $\lambda \sim R_{so}$.

In running the 3D simulation on a Cartesian grid, we found an interesting phenomenon. The animation shown in Figure 10 illustrates the initial effects of the Cartesian grid. The first ~ 20 flow times are characterized by variations in the bowshock shape that remain highly symmetric. As the shock extends in z it collapses in y, and vice-versa, maintaining a four-fold symmetry. However, after approximately 23 flow times the evolution appears to make a relatively abrupt change, losing all sense of symmetry within only a few flow times. We suggest that the latter half of the simulation is more likely to be representative of the true evolution of isothermal bowshocks. This simulation emphasizes the importance of initial conditions and the need to evolve simulations for many flow times. Even after 20 flow times the geometry of the bowshock is evidently still strongly influenced by the initial conditions.

3.5 Discussion

Isothermal stellar wind bowshocks are clearly unstable. The hydrodynamic simulations shown here dramatically illustrate this fact. In addition to the requirement of rapid radiative cooling (such that the post-shock cooling length is much less than the standoff distance of the bowshock), the instability depends primarily on the Mach number of the star moving through the ISM, requiring a minimum Mach number of a few. The effect of the instability is to ripple the bowshock with wavelengths and amplitudes of order the nominal standoff distance of the bowshock, R_{so}.

The most likely sources for unstable isothermal bowshocks would be stars with slow, dense winds, such as red supergiants and AGB stars, moving through the ISM with speeds of order $60\,\mathrm{km\,s^{-1}}$. The high gas density and low velocity of the cool winds from these stars can lead to rapid cooling behind the reverse shock. Given a typical ISM sound speed of $\sim 15\,\mathrm{km\,s^{-1}}$, a space velocity of greater than $\sim 45\,\mathrm{km\,s^{-1}}$ would be needed to drive the instability. Coincidentally, shock velocities not much larger than this (in the neighborhood of 60 to 80 $\mathrm{km\,s^{-1}}$) have relatively large cooling rates and correspondingly small postshock cooling lengths (Shull & McKee 1979). Thus a space velocity of order $70\,\mathrm{km\,s^{-1}}$ would favor rapid cooling behind the leading bowshock and provide a sufficiently large Mach number to drive this instability.

Betelgeuse may be an example of a star with an unstable bowshock. This red supergiant exhibits an asymmetric circumstellar shell (perhaps a bowshock) at a standoff distance of order 1 pc (Noriega-Crespo et al. 1997). These authors estimate a peculiar velocity for Betelgeuse of $\sim 56\,\mathrm{km\,s^{-1}}$ and a wind velocity of $\sim 18\,\mathrm{km\,s^{-1}}$. These velocities are low enough that the postshock cooling length will be smaller than the shell thickness, particularly if cooling is enhanced by molecular emission. However, the relatively low-resolution images presented by Noriega-Crespo et al. (1997) do not show evidence for an instability. This apparent lack of instability could be explained either by a low Mach number of Betelgeuse moving through the ISM or by a cooling rate that is sufficiently slow that the isothermal approximation is not valid. Noriega-Crespo et al. (1997) derive an ambient density of $\sim 0.1\mathrm{cm}^{-3}$. If this ambient material is warm, ionized gas, the Mach number through the ISM may be only of order a few - perhaps insufficient to strongly excite the instability discussed in this paper.

Based on the internal dynamics of the slab, the overall evolution of bends in the slab, and the fact that the instability grows fastest near the apex of the bowshock, we argue that the instability of isothermal stellar wind bowshocks can be attributed to the NTSI. The NTSI is driven by shear flows within the shell created by large-wavelength wiggles in the shell. Once the large distortions created by the NTSI advect into the wings of the bowshock, the action of the TAI becomes important in further distorting the shape of the bowshock. The bending mode of the TAI originates in the unequal momentum flux delivered to the shell when the shell orientation is rotated from the equilibrium position as shown in Figure 4.

The instability studied here is clearly seen in the numerical simulations described in Brighenti & D'Ercole (1995). These authors argue that the dramatic swings of the bowshock are primarily a result of Kelvin-Helmholtz and Raleigh-Taylor instabilities. They apparently dismiss the role of the NTSI on the grounds that they are witnessing nonlinear oscillations - but NTSI stands for *Nonlinear* Thin-Shell Instability! Instead they argue that the KH instability is more relevant in the bowshock problem as compared to the planar slab analysed by Vishniac (1994) because the unperturbed planar slab has no shear, while the unperturbed bowshock does (although it is quite small near the apex - the region of maximum growth - for $v_w = v_*$). But, as Brighenti & D'Ercole point out, they are witnessing nonlinear displacements, and in that nonlinear regime, the interior shear is determined not by the unperturbed solution but by the nonlinear bends of the bowshock. In this case the role of the KH instability should be no different in the bowshock case or the planar slab case. The simulations of Blondin & Marks (1997) show that the KH instability is indeed operating within the unstable slab, but it does not strongly affect the growth of the NTSI.

Finally, we note that although strong cooling behind both the forward and reverse shocks is a requirement of the isothermal instability studied here, it may not be a necessary condition for an unstable bowshock. If both shocks are adiabatic the bowshock is expected to be stable (although the contact discontinuity is subject to the Kelvin-Helmholtz instability), but if one or the other of the two shocks are effectively isothermal the bowshock may be unstable. Raga et al. (1997) ran simulations of a bowshock around a runaway O star with parameters such that the reverse shock was adiabatic owing to the high wind velocity and the forward shock was radiatively cooling. They showed that indeed the shell of cooled gas behind the leading shock is unstable. It would not be unreasonable to expect a similar instability in the opposite case of an isothermal reverse shock and an adiabatic forward shock. Such a situation might arise, for example, in the case of an AGB star travelling rapidly ($v_* > 100\,\mathrm{km\,s^{-1}}$) through the ISM.

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Fig. 1. Schematic of the idealized stellar wind bowshock. A uniform flow approaches the bowshock from the right and a spherically symmetric wind emanates from the position of the star. The heavy line is the equilibrium bowshock solution of Wilkin (1996). The coordinates are measured in units of the standoff distance R_{so}.

Fig. 2. Snapshot in time from the high-resolution simulation of an unstable isothermal bowshock with $M = 10$. The full time evolution of this unstable bowshock is shown in the accompanying animation. The shading represents the gas density, with black corresponding to the highest density. The solid line marks the analytic solution for the shape of a thin bowshock as given by Wilkins (1997).

Fig. 3. The flow velocity within a section of the unstable bowshock shown in Figure 2. The ISM flows uniformly in from the right while the stellar wind, with some spherical divergence evident, flows in from the left. The solid lines mark the inner and outer shocks bounding the bowshock shell. The shear inside the shell is similar to that seen in the NTSI, with postshock flow directed towards extrema in the perturbed slab.

Fig. 4. The departure of the shell orientation in the center of this image from the equilibrium solution (shown as a solid line) results in the stellar wind impacting the shell almost head on at this location, while the ISM flow barely grazes the surface of the shell. The result is a gross imbalance of the momentum delivered to the shell, driving it away from the star. This is the Transverse Acceleration Instability.

Fig. 5. Dependence of bowshock evolution on numerical resolution. The three images correspond to low (120^2), medium (240^2), and high (480^2) resolution simulations of a Mach $= 5$ simulation. Although there is more small scale structure in the higher resolution simulations, all three look qualitatively similar, with the same dominant wavelength of $\lambda \sim R_{so}$.

Fig. 6. The bowshock instability increases with increasing Mach number of the stellar wind and ISM flow. The Mach numbers are, from left to right, 2.5, 5, and 10. The role of the instability as a function of Mach number is best seen in the accompanying animations.

Fig. 7. Bowshocks are more unstable for large v_* than for large v_w. The image on the left is from a simulation with $v_w = 8$ and $v_* = 2$ ($\beta = 4$), and on the right from a simulation with $v_w = 2$ and $v_* = 8$ ($\beta = 0.25$). The evolution of these bowshocks is shown in the accompanying animations.

Fig. 8. Comparison of bowshock evolution in two (right) and three (left) dimensions.

Fig. 9. An isodensity surface showing the leading surface of the bowshock in the three-dimensional simulation. The onrushing ISM is approaching the bowshock from above, and the source of the stellar wind is buried in the center of this image at coordinates (0,0,0). The accompanying animation shows a short time segment in the evolution of this bowshock surface.

Fig. 10. Effects of initial conditions on a Cartesian grid are evident in this slice through the three-dimensional simulation. Black corresponds to the high density gas in the bowshock shell. The times for these images are, from left to right, 14.7, 23.2, and 34.3, in units of the flow time R_{so}/v_w. The accompanying animation shows the full time evolution. Early in the evolution (left image) the bowshock exhibits clear symmetry. Around $t \sim 23R_{so}/v_w$ this symmetry quickly disappears.