4-6) A gold foil of thickness 2.0 μm is used in a Rutherford experiment to scatter α particles with energy 7.0 MeV. (a) What fraction of the particles will be scattered at angles greater than 90 degrees? (b) What fraction will be scattered at angles between 45 and 75 degrees? [Answer: 9.8×10⁻⁵; 4.05×10⁻⁴]

The fraction of incident particles scattered through an angle greater than θ is given by

\[ f = (1 + b)^n \] where \[ b = \frac{kq_x q_y}{Z \times E_x} \]

The column density for 2 μm-thick gold foil is

\[ n_t = \frac{\text{mass density} \times \text{Avogadro's number} \times \text{thickness}}{\text{Molar mass}} \]

\[ = (19.3 \text{ g/cm}^2)(6.02 \times 10^{23} \text{ atoms/mole})(\frac{1 \text{ mole}}{197 \text{ g}})(2 \times 10^{-4} \text{ cm}) = 1.18 \times 10^{19} \text{ atoms/cm}^2 \]

The impact parameter for α particles on gold nuclei is

\[ b = k \frac{(2e)(79e)}{2 \times (7 \text{ MeV})} \cot \frac{\theta}{2} = 79 \frac{k e^2}{7 \text{ MeV}} \cot \frac{\theta}{2} = 79 \frac{1.44 \text{ eV nm}}{7 \times 10^{-6} \text{ eV}} \cot \frac{\theta}{2} \]

\[ b = (1.62 \times 10^{-12} \text{ cm}) \cot \frac{\theta}{2} \]

a) For θ = 90°, \[ \cot \frac{\theta}{2} = 1 \] and \[ f = n_t (1.62 \times 10^{-12} \text{ cm}) (1.18 \times 10^{19} \text{ cm}^{-2}) \]

\[ f = 9.73 \times 10^{-5} \]

b) For fraction between angles of 45° and 75°

\[ f = f_{45} - f_{75} = n_t (n_t) \left( b_{45}^2 - b_{75}^2 \right) \]

\[ = n_t (1.18 \times 10^{19} \text{ cm}^{-2}) (1.62 \times 10^{-12} \text{ cm}) \left( (\cot 22.5°)^2 - (\cot 37.5°)^2 \right) \]

\[ = 9.73 \times 10^{-5} \left[ 2.414^2 - 1.303^2 \right] \]

\[ f = 4.02 \times 10^{-4} \]
What will be the distance of closest approach to a gold nucleus for an α particle of energy 5.0 MeV? 7.7 MeV? 12 MeV? [Answer: 45.5 fm]

\[ q = +2e \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad q = +79e \]

\[ \text{Use conservation of energy: } E = \frac{1}{2} m v^2 + \frac{1}{d} \frac{kq^2}{d} = \text{constant} \]

At the source, d is very large and total energy is just the kinetic energy of the α particle. At closest approach in a head-on collision, the α has stopped and will start moving backwards. At this moment the kinetic energy is zero and all energy is in the electrostatic potential. Conservation of energy implies

\[ E_0 = \frac{1}{2} M α v_0^2 = 5 \text{ MeV} = \frac{kq^2}{d_0} \quad \text{where } d_0 \text{ is distance of closest approach} \]

\[ d_0 = \frac{Z_2 Z_A (\text{ke}^2)}{E_0} = \frac{158 \times 1.144 \text{ eV} \cdot \text{nm}}{5 \times 10^6 \text{ eV}} = 4.55 \times 10^{-5} \text{ nm} = 45.5 \text{ fm} \]

<table>
<thead>
<tr>
<th>( E_0 )</th>
<th>( d_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 MeV</td>
<td>45.5 fm</td>
</tr>
<tr>
<td>7.7 MeV</td>
<td>29.5 fm</td>
</tr>
<tr>
<td>12 MeV</td>
<td>19.0 fm</td>
</tr>
</tbody>
</table>
4-15) Calculate the three longest wavelengths in the Lyman series in nm, and indicate their position on a horizontal wavelength scale. Indicate the series limit (shortest wavelength) on this scale. Are any of these lines in the visible spectrum? [Answer: 121.6 nm]

The Lyman series corresponds to transitions to/from the n=1 energy level. First find the change in energy of these transitions, and then use \( \lambda = \frac{hc}{E} \) to compute the wavelengths.

\[ E_\infty = 0 \]
\[ E_4 = -0.85 \text{ eV} \]
\[ E_3 = -1.51 \text{ eV} \]
\[ E_2 = -3.40 \text{ eV} \]

3 longest wavelengths correspond to the 3 smallest energy changes

<table>
<thead>
<tr>
<th>Transition</th>
<th>( \Delta E )</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n=2 \rightarrow 1 )</td>
<td>(-3.40 eV) - (-13.60 eV) = 10.20 eV</td>
<td>( \frac{10.20 \text{ eV}}{121.5 \text{ nm}} \approx 97 \text{ nm} )</td>
</tr>
<tr>
<td>( n=3 \rightarrow 1 )</td>
<td>(-1.51 eV) - (-13.60 eV) = 12.09 eV</td>
<td>( \frac{12.09 \text{ eV}}{102.5 \text{ nm}} \approx 119 \text{ nm} )</td>
</tr>
<tr>
<td>( n=4 \rightarrow 1 )</td>
<td>(-0.85 eV) - (-13.60 eV) = 12.75 eV</td>
<td>( \frac{12.75 \text{ eV}}{97.25 \text{ nm}} \approx 132 \text{ nm} )</td>
</tr>
<tr>
<td>( n=\infty \rightarrow 1 )</td>
<td>(0) - (-13.60 eV) = 13.60 eV</td>
<td>( \frac{13.60 \text{ eV}}{91.17 \text{ nm}} \approx 121 \text{ nm} )</td>
</tr>
</tbody>
</table>

\[ \lambda_{2\rightarrow 1} = \frac{123.9 \text{ eV} \cdot \text{nm}}{10.20 \text{ eV}} \approx 121.5 \text{ nm} \]

Visible begins around 390 nm
4-19) It is possible for a muon to be captured by a proton to form a muonic atom. A muon is identical to an electron except for its mass, which is 105.7 MeV/c². (a) Calculate the radius of the first Bohr orbit of a muonic atom. (b) Calculate the magnitude of the lowest energy. (c) What is the shortest wavelength in the Lyman series for this atom? [Answer: $2.56 \times 10^{-4}$ nm; 2.52 keV; 0.492 nm]

The Bohr radius is \[ \frac{\hbar^2}{m_e ke} = 0.0529 \text{ nm} \]

For the muonic atom the mass of the electron is replaced by the mass of the muon, 105.7 MeV/c². The muon, however, is much more massive than the electron. As a result, it is no longer reasonable to assume the nucleus is fixed in space. To account for the motion of the nucleus, we use the reduced mass of the muon

\[ M_m = \frac{m_\mu}{1 + m_\mu/m_p} = \frac{105.7 \text{ MeV}/c^2}{1 + \frac{105.7}{938.3}} = 95.0 \text{ MeV} \]

a) radius of first Bohr orbit \[ r = \frac{\hbar^2}{M_m ke} = \frac{m_e}{M_m} a_0 \]

\[ r = \frac{0.511 \text{ MeV}}{95.0 \text{ MeV}} (0.0529 \text{ nm}) \]

b) energy of ground state \[ E_0 = -\frac{m_k e^4}{2\hbar^2} \left( \frac{M_m}{m_e} \right)(-13.60 \text{ eV}) = 2.53 \text{ keV} \]

c) shortest wavelength (largest energy) of Lyman series is a transition from n=∞ ($E=0$) to n=1 ($E=2.53 \text{ keV}$)

\[ \lambda = \frac{hc}{E} = \frac{1239.8 \text{ eV} \text{ nm}}{2.528 \text{ eV}} = 0.49 \text{ nm} \]
4-27) The wavelength of the $K_{\alpha}$ x-ray line for an element is measured to be 0.0794 nm. What is the element?

$K_{\alpha}$ line is created when an $e^{-}$ falls from $n=2$ to $n=1$, but nucleus is partially shielded by one $e^{-}$ already in $n=1$ state. So effective nuclear charge is $Z-1$:

$$\frac{1}{\lambda} = R \left( \frac{Z-1}{1^2} - \frac{1}{2^2} \right) \Rightarrow \frac{1}{\lambda} = \frac{\hbar}{3 \lambda R} = \frac{4 \hbar}{3 (0.0794 \text{ nm})(1.097 \times 10^{-2} \text{ nm})}$$

Element $Z=610$ is $\text{Zr}$

Zirconium

5-3) Electrons in an electron microscope are accelerated from rest through a potential difference $V_0$ so that their de Broglie wavelength is 0.04 nm. What is $V_0$?

The de Broglie wavelength is given by $\lambda = \frac{\hbar}{p}$. If the speed is non-relativistic, the kinetic energy of the electron ($eV_0$) is related to its momentum by $E = \frac{p^2}{2m}$

$$Zme(eV_0) = \frac{p^2}{2m} = \frac{\hbar^2}{4\pi^2}$$

$$eV_0 = \frac{(\hbar c)^2}{2(511,000 \text{ eV})(0.04 \text{ nm})^2} = 940 \text{ eV}$$

$$\Rightarrow V_0 = \frac{940 \text{ eV}}{940} = 1 \text{ Volts}$$

Since kinetic energy is much less than rest energy, 940 eV << 511 keV, $e^{-}$ is indeed non-relativistic.

5-6) Find the de Broglie wavelength of a neutron of kinetic energy 0.02 eV (this is of the order of magnitude for $kT$ at room temperature). [Answer: 0.202 nm]

At room temperature a neutron will have a typical kinetic energy of $kT = \frac{1.38 \times 10^{-23} \text{ J/K}}{1.6 \times 10^{-19} \text{ J/eV}} = 8.59 \text{ eV}$.

Since this energy is $\ll mc^2$, neutron is non-relativistic and $E = \frac{p^2}{2m}$. Using $\lambda = \frac{\hbar}{p}$,

$$\lambda = \frac{\hbar}{2mE_k} = \frac{\hbar c}{2m^2 E_k} = \frac{(1239.8 \text{ eV nm})}{(2 \times 939.5 \times 10^6 \text{ eV} \times 0.02 \text{ eV})} = 0.202 \text{ nm}$$