

7-36) A hydrogen atom is in the 3D state ($n=3, l=2$). (a) What are the possible values of j ? (b) What are the possible values of the magnitude of the total angular momentum? (c) What are the possible z components of the total angular momentum?

Add spin and orbital angular momentum together using the addition rule in section 7-5:

$$j = l+s, l+s-1, \dots, |l-s| \Rightarrow j = l \pm \frac{1}{2}$$

where $l=2$ and $s=\frac{1}{2}$, so $j = \underline{\underline{\frac{5}{2} \text{ or } \frac{3}{2}}}$

The total angular momentum is $|\vec{J}| = \sqrt{j(j+1)} \hbar$

$$|\vec{J}| = \sqrt{\frac{5}{2}\left(\frac{7}{2}\right)} \hbar = 2.86 \hbar$$

$$\text{or } \sqrt{\frac{3}{2}\left(\frac{5}{2}\right)} \hbar = 1.94 \hbar$$

For $j = \frac{5}{2}$, the possible values of m_j are

$$-5/2, -3/2, -1/2, +1/2, +3/2, +5/2 \text{ with } J_z = m_j \hbar$$

and for $j = \frac{3}{2}$, the possible values of m_j are

$$-3/2, -1/2, +1/2, +3/2$$

7-39) Consider a system of two electrons, each with $l=1$ and $s=1/2$. (a) What are the possible values of the quantum number for the total orbital angular momentum $L=L_1+L_2$? (b) What are the possible values of the quantum number S for the total spin $S=S_1+S_2$? (c) Using the results above, find the possible quantum numbers j for the combination $J=L+S$. (d) What are the possible quantum numbers j_1 and j_2 for the total angular momentum of each particle? (e) Use the results of part (d) to calculate the possible values of j from the combinations of j_1 and j_2 . Are these the same as in part (c)?

a) The quantum number for the total orbital angular momentum is given by the angular momentum addition rule given in section 7-5, namely l can have the values

$$l_1 + l_2, l_1 + l_2 - 1, \dots, |l_1 - l_2|$$

Here the maximum is $1+1=2$, the minimum is $1-1=0$, so l has possible values of 2, 1, or 0.

b) The same rule applies to total spin

$s_1 + s_2 = \frac{1}{2} + \frac{1}{2} = 1$, minimum $s_1 - s_2 = 0$, and no integers in between, so s has possible values of 1 and 0.

c) Apply the same rule when adding $\vec{J} = \vec{L} + \vec{S}$

l	s	j
2	1	3, 2, 1
	0	2
1	1	2, 1, 0
	0	1
0	1	1
	0	0

$$j = l + s, l + s - 1, \dots, |l - s|$$

possible values of j are 0, 1, 2, or 3.

d) possible values of j_1 are

$$(l_1 + s_1), \dots, |l_1 - s_1|$$

$$3/2, 1/2 \text{ (same for } j_2 \text{)}.$$

e) find j by adding \vec{J}_1 and \vec{J}_2 using $j = j_1 + j_2, |j_1 - j_2|, \dots, |j_1 - j_2|$

for $j_1 = j_2 = 3/2$, $j = \underline{3, 2, 1, \text{ or } 0}$

$j_1 = j_2 = 1/2$, $j = 1 \text{ or } 0$

$j_1 = 3/2, j_2 = 1/2$ $j = 2, 1$

$= 1/2 = 3/2$ $j = 2, 1$

3, 2, 1, or 0

is the same

7-40) The prominent yellow doublet lines in the spectrum of sodium result from transitions from the $3P_{3/2}$ and $3P_{1/2}$ states to the ground state. The wavelengths of these two lines are 589.6 nm and 589.0 nm. (a) Calculate the energies in eV of the photons corresponding to these wavelengths. (b) The difference in energy of these photons equals the difference in energy of the $3P_{3/2}$ and $3P_{1/2}$ states. This energy difference is due to the spin-orbit effect. Calculate this energy difference. (c) If the $3p$ electron in sodium sees an internal magnetic field B , the spin-orbit energy splitting will be of the order of $\Delta E = 2\mu_B B$, where μ_B is the Bohr magneton. Estimate B from the energy difference ΔE found in part (b).

Photon energies are given by $E = \frac{hc}{\lambda}$

$$589.6 \text{ nm} \rightarrow E_1 = \frac{1239.8 \text{ eV nm}}{589.6 \text{ nm}} = \underline{2.105 \text{ eV}}$$

$$E_2 = \frac{1239.8 \text{ eV nm}}{589.0 \text{ nm}} = \underline{2.103 \text{ eV}}$$

The difference in energy between these two photons is $\Delta E = 2.105 - 2.103 = 2 \times 10^{-3} \text{ eV}$

But this is influenced by our rounding, so let's calculate ΔE directly

$$\begin{aligned} \Delta E &= E_2 - E_1 = hc \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right) = \frac{hc}{\lambda_1 \lambda_2} (\lambda_1 - \lambda_2) \\ &= \frac{1239.8 \text{ eV nm}}{(589.6 \text{ nm})(589.0 \text{ nm})} (0.6 \text{ nm}) = \underline{2.14 \times 10^{-3} \text{ eV}} \end{aligned}$$

If this ΔE is attributed to a magnetic interaction, the effective magnetic field is

$$B = \frac{\Delta E}{2\mu_B} = \frac{2.1 \times 10^{-3} \text{ eV}}{2 \times 5.8 \times 10^{-4} \text{ eV/T}} = \underline{\underline{1.8 \text{ T}}}$$

7-43) Five identical non-interacting particles are placed in an infinite square well with $L=1.0\text{nm}$. compute the lowest total energy for the system if the particles are (a) electrons and (b) pions. Pions have symmetric wave functions and their mass is $264m_e$.

The energy levels of a particle in an infinite, one-dimensional square well are given by

$$E_n = n^2 \frac{hc^2}{8L^2mc^2} = n^2 E_1$$

a) Electrons are Fermions, and will obey the Pauli exclusion principle. This means only two electrons can occupy the ground state, E_1 , one electron with spin 'up', the other with opposite spin. The lowest energy state

$$\begin{aligned} \text{thus has } 2 \text{ in state } n=1 & \quad E = 2 \times E_1 \\ & \quad 2 \text{ in state } n=2 \quad E = 2 \times 4E_1 \\ & \quad 1 \text{ in state } n=3 \quad E = 1 \times 9E_1 \\ & \quad \text{total } E = 19E_1 \end{aligned}$$

$$E_1 = \frac{(1239.8 \text{ eV nm})^2}{8(511,000 \text{ eV})(1.0 \text{ nm})^2} = 0.376 \text{ eV}$$

$$\text{total energy } \underline{E = 7.14 \text{ eV}}$$

b) Pions are bosons, which do not obey Pauli exclusion. Lowest energy for the system is when all five pions are in the $n=1$ state

$$\underline{E = 5E_1 = 5 \frac{(1239.8 \text{ eV nm})^2}{8(10 \text{ nm})^2(264 \times 511,000 \text{ eV})} = 0.00712 \text{ eV}}$$

7-47) Which of the following atoms would you expect to have its ground state split by the spin-orbit interaction: Li, B, Na, Al, K, Ag, Cu, Ga? (Hint: Use Appendix C to see which elements have $l=0$ in their ground state and which do not.)

The spin-orbit interaction requires electron spin, which all single electrons have, but electrons paired up in a state have zero spin, and orbital angular momentum.

Li $1s^2 2s$

B $1s^2 2s^2 2p^1 \leftarrow$

Na $1s^2 2s^2 2p^6 3s$

Al $1s^2 2s^2 2p^6 3s^2 3p^1 \leftarrow$

K $1s^2 2s^2 2p^6 3s^2 3p^6 4s$

Ag $1s^2 \dots 4p^6 4d^{10} 5s$

Cu $1s^2 \dots 3p^6 3d^{10} 4s$

Ga $1s^2 \dots 3p^6 3d^{10} 4s^2 4p^1 \leftarrow$

Li, Na, K, Ag, Cu all have $l=0$ (s-state) for the outermost, unpaired electron. With no net l , there is no spin-orbit coupling.

Only B, Al, Ga have outermost, unpaired electrons with $l \neq 0$. These three atoms will have their ground state split.