1) A particle of mass 1.00 MeV/c² and kinetic energy 2.00 MeV collides with a stationary particle of mass 2.00 MeV/c². After the collision, the particles stick together.

A. Find the momentum and total energy of this two-particle system before the collision.

\[ m_1 = 1 \quad \quad m_2 = 2 \]

\[ p \rightarrow 0 \]

Total energy of particle \#1 = \( mc^2 \) + kinetic = \( 1 + 2 = 3 \) MeV

Find momentum of particle \#1 using \( (pc)^2 = E^2 - (mc^2)^2 \)

\[ (pc)^2 = 9 \text{ MeV}^2 - 1 \text{ MeV}^2 = 8 \text{ MeV}^2 \Rightarrow p = \sqrt{8} \text{ MeV/c} \]

Total energy of particle \#2 is just rest mass = 2 MeV

momentum of particle \#2 is zero

Add it all up:

\[ E = 3 + 2 = 5 \text{ MeV} \]

\[ p = \sqrt{8} + 0 = \sqrt{8} \text{ MeV/c} = 2.83 \text{ MeV/c} \]

B. What is the rest mass energy of the composite particle formed in the collision?

\[ E \text{ and } p \text{ remain the same, but now there is just one} \]

particle with unknown mass \( M \) that must satisfy

\[ (Mc^2)^2 = E^2 - (pc)^2 = 25 \text{ MeV}^2 - 8 \text{ MeV}^2 = 17 \text{ MeV}^2 \]

\[ M = \sqrt{17} \text{ MeV/c}^2 = 4.12 \text{ MeV/c}^2 \]

C. What is the kinetic energy of the composite particle after the collision?

\[ \text{Kinetic energy = total energy - rest mass energy} \]

\[ = 5 \text{ MeV} - \sqrt{17} \text{ MeV} \]

\[ = 5 - 4.12 = 0.88 \text{ MeV} \]
2) A particle of rest mass 1.00 MeV/c^2 is moving with a speed of 0.8c when it collides with its antiparticle which is at rest in this observer frame. The annihilation creates two photons. What are the energies of these two photons in this observer frame?

A. What are the energies and momenta of the particles in the observer frame S?

particle: \( E_1 = \gamma mc = 5\sqrt{3} \text{ MeV} \quad p_1 = \gamma mu = x/\beta mc = 5/3 \text{ m} \text{eV}/c \)

antiparticle: \( E_2 = 1 \text{ MeV} \quad p_2 = 0 \)

B. Transform the momenta to a frame \( S' \) moving relative to S with an arbitrary speed \( \beta \) in a direction along the motion of the original particle.

\[
\begin{align*}
p'_1 &= \gamma_T \left(p_1 - \beta E/c\right) = \gamma_T \left(\frac{4}{3} - \frac{\sqrt{3}}{3}\right) \\
p'_2 &= \gamma_T \left(p_2 - \beta E/c\right) = -\gamma_T \beta
\end{align*}
\]

C. Find the speed \( \beta_c \) of the frame \( S' \) for which the momenta of the particle and antiparticle will be equal and opposite (the center of momentum frame).

\[
\begin{align*}
\text{in } S' \quad \text{want } p'_1 + p'_2 &= 0, \quad \text{so} \\
\gamma_c \left(\frac{4}{3} - \frac{\sqrt{3}}{3}\right) &= \gamma_c \beta_c \\
\frac{4}{3} &= \left(1 + \frac{\sqrt{3}}{3}\right) \beta_c \\
\beta_c &= 0.6 \\
\gamma_c &= \frac{4}{3}
\end{align*}
\]

D. Using this value of \( \beta_c \), compute the energy of each particle in this center of momentum frame \( S_c \). Energies will be the same, so easiest to transform particle 2:

\[
\begin{align*}
E'_2/c &= \gamma_c (E_2/c - \beta_c P) = \sqrt{\frac{4}{3}} (1) = \sqrt{\frac{4}{3}} \\
E'_2 &= E'_2' c = \sqrt{\frac{4}{3}} \text{ MeV}
\end{align*}
\]

E. What is the energy of each photon in \( S_c \), after the annihilation?

\[
\begin{align*}
\text{each photon will have the same energy as one of particles} \\
E_{\text{photon}} &= \sqrt{\frac{4}{3}} \text{ MeV}
\end{align*}
\]

F. Transform these photon energies back to the original observer frame S.

\[
\text{photon going to the right:} \quad E'/c = \gamma_c (E'/c + \beta_c P) \\
\begin{align*}
E' &= \sqrt{\frac{4}{3}} \left(\sqrt{\frac{4}{3}} + 0.5 \sqrt{\frac{4}{3}}\right) \\
E' &= \frac{4}{3} \times 3/2 = 2 \text{ MeV}
\end{align*}
\]

\[
\text{photon going to the left:} \quad E'/c = \gamma_c (E - \beta_c P) \\
\begin{align*}
E' &= \sqrt{\frac{4}{3}} \left(\sqrt{\frac{4}{3}} - 0.5 \sqrt{\frac{4}{3}}\right) \\
E' &= \frac{4}{3} \times 3/2 = \frac{2}{3} \text{ MeV}
\end{align*}
\]
3) A Klingon Battle Cruiser is equipped with a disruptor cannon mounted on its rear end which can only fire perpendicularly to its direction of motion. A standard Klingon tactic is to fly by its enemy and fire this cannon as the front end of the Battle Cruiser passes the enemy ship's rear end. In the following discussion, assume that the Battle Cruiser flies by so closely that you can neglect the time it takes for the disruptor beam to travel from the cannon to the target.

The Starship Enterprise, which is the same length as the Battle Cruiser when both are at rest, is about to be attacked. Dr. McCoy is worried that, due to Lorentz contraction, the Klingon Battle Cruiser will be shorter than the Enterprise and therefore cannot miss. Captain Kirk, on the other hand, claims that the Enterprise will be shorter than the Battle Cruiser when seen from the Klingons' frame so that there is nothing to worry about.

Answer the following questions:

A. At which space-time point (labeled A through H), does the front end of the Battle Cruiser pass by the rear end of the Enterprise?
B. As the front end of the Battle Cruiser passes by the rear end of the Enterprise, at which space-time point is the rear end of the Battle Cruiser in the Enterprise's frame?
C. As the front end of the Battle Cruiser passes by the rear end of the Enterprise, at which space-time point is the front end of the Enterprise in the Battle Cruiser's frame?
D. At which space-time point does the disruptor cannon fire?
E. When the disruptor cannon fires, at which space-time point is the front end of the Battle Cruiser in the Enterprise frame?
F. Explain to Captain Kirk and Dr. McCoy which of them is correct. Refer to the space-time diagram in your explanation.

Kirk is correct, the cannon is fired simultaneously with event C as defined in the Klingon's frame.