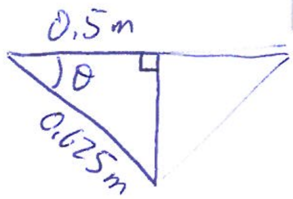
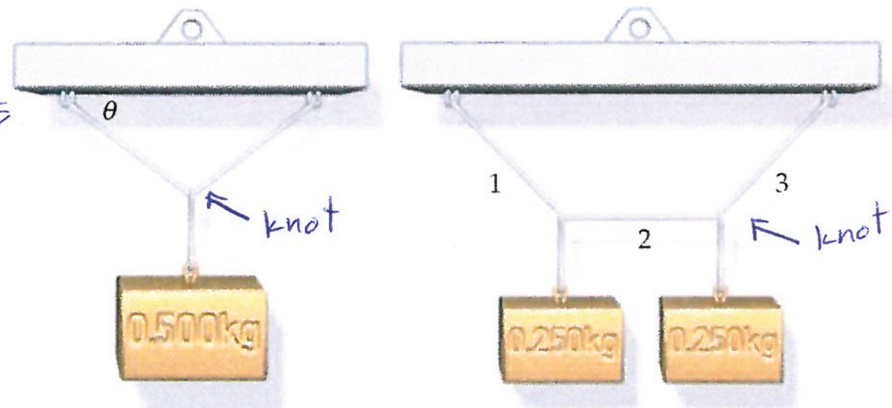
PY 205H – Homework #2

4-49) In the figure on the left, a 0.500-kg block is suspended at the midpoint of a 1.25-m-long string. The ends of the string are attached to the ceiling at points separated by 1.00 m. (*a*) What angle does the string make with the ceiling? (*b*) What is the tension in the string? (*c*) The 0.500-kg block is removed and two 0.250-kg blocks are attached to the string such that the lengths of the three string segments are equal (right figure). What is the tension in each segment of the string?
[$\theta = 37^o$, $T = 4.1N$, $T_1 = T_3 = 3.4N$, $T_2 = 2.4N$]

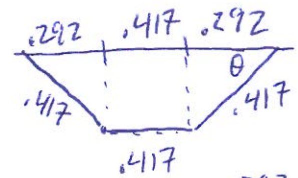


$$\cos\theta = \frac{0.5}{0.625}$$

a) $\boxed{\theta = 37^\circ}$



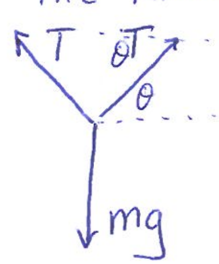
(a) (b)



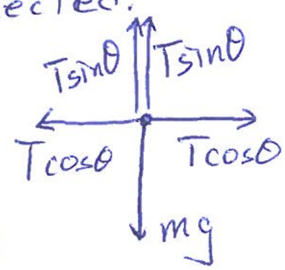
$$\cos\theta = \frac{.292}{.417} = .7$$

$$\theta = 46^\circ$$

b) Find the tension in the string by applying $\sum \vec{F} = m\vec{a}$ to the knot where the strings are connected.



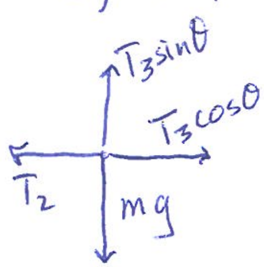
Break $\vec{T}$ into horizontal and vertical components



Since there is no motion, $a=0$, and $\sum F_x = 2T\sin\theta - mg = 0$

$$T = \frac{mg}{2\sin\theta} = \frac{(0.5\ kg)(9.8\ m/s^2)}{2 \cdot \sin 37^o} = \boxed{4.1\ N}$$

c) Repeat for knots in second problem

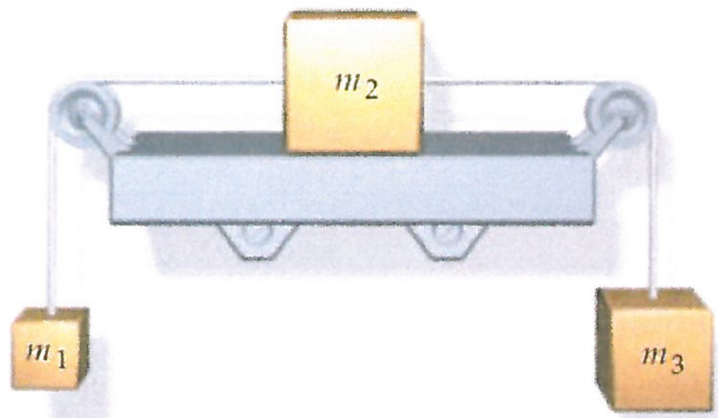


vertical: $T_3 \sin\theta = mg \quad T_3 = \frac{mg}{\sin\theta} = \frac{(0.25\ kg)(9.8\ m/s^2)}{\sin 46^o} = \boxed{3.4\ N}$

horizontal: $T_2 = T_3\cos\theta = \frac{mg}{\sin\theta}\cos\theta = \frac{mg}{\tan\theta} = \boxed{2.4\ N}$

From the symmetry of the problem, $T_1 = T_3 = 2.4N$

4-67) A box of mass $m_2 = 3.5$ kg rests on a frictionless horizontal shelf and is attached by strings to boxes of masses $m_1 = 1.5$ kg and $m_3 = 2.5$ kg as shown in the figure below. Both pulleys are frictionless and massless. The system is released from rest. After it is released, find (*a*) the acceleration of each of the boxes, and (*b*) the tension in each string. [$a = 1.3\ \text{m/s}^2$, $T_1 = 17$N, $T_3 = 21$N]



All 3 boxes will move with the same acceleration. Solve $F = ma$ for each of the boxes, using clockwise as the positive direction. $\uparrow \rightarrow \downarrow$

Box 1 $\quad T_1,\ m_1 g \qquad T_1 - m_1 g = m_1 a \qquad T_1 = m_1 g + m_1 a$

$T_2 = m_3 g - m_3 a$

Box 2 $\quad T_1,\ T_2 \qquad T_2 - T_1 = m_2 a \qquad (m_3 g - m_3 a) - (m_1 g + m_1 a) = m_2 a$

$(m_3 - m_1) g = a\,(m_1 + m_2 + m_3)$

Box 3 $\quad T_2,\ m_3 g \qquad m_3 g - T_2 = m_3 a$

$$a = \frac{m_3 - m_1}{m_1 + m_2 + m_3}\, g$$

$$= \frac{2.5 - 1.5}{1.5 + 2.5 + 3.5} \times 9.8\ \text{m/s}^2$$

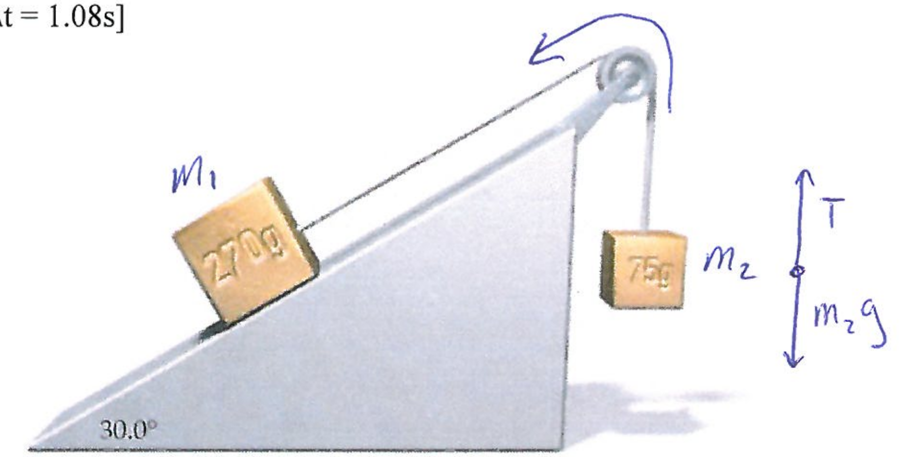
$$\boxed{a = 1.3\ \text{m/s}^2}$$

Use a to find Tensions

$T_1 = m_1(g + a) = (1.5\ \text{kg})(9.8 + 1.3) = \boxed{17\ \text{N}}$

$T_2 = m_3(g - a) = (2.5\ \text{kg})(9.81 - 1.3) = \boxed{21\ \text{N}}$

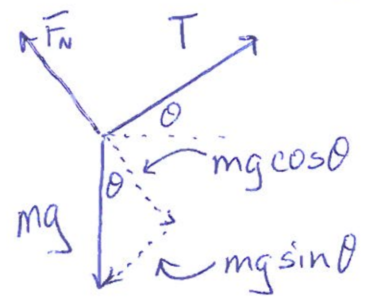
4-90) A frictionless surface is inclined at an angle of 30.0° to the horizontal. A 270-g block on the ramp is attached to a 75.0-g block using a pulley, as shown in the figure below. (*a*) Draw two free-body diagrams, one for the 270-g block and the other for the 75.0-g block. (*b*) Find the tension in the string and the acceleration of the 270-g block. (*c*) The 270-g block is released from rest. How long does it take for it to slide a distance of 1.00 m along the surface? Will it slide up the incline, or down the incline?
[T = 0.864N, *a* = 1.71 m/s², Δt = 1.08s]



Forces on hanging block: Tension of rope, gravity

choosing up as positive direction: $\sum \vec{F} = T - m_2 g = m_2 a$

Forces on sliding block: Tension, gravity, normal force



along ramp: $mg\sin\theta - T = m_1 a$ (down ramp is positive)

⊥ to ramp: $F_N - mg\cos\theta = 0$

Combine $T = m_2 a + m_2 g$
$T = m_1 g\sin\theta - m_1 a$ $\Rightarrow$ $m_2 a + m_2 g = m_1 g(\tfrac{1}{2}) - m_1 a$

$(m_1 + m_2)a = (\tfrac{1}{2}m_1 - m_2)g$

$a = \dfrac{\tfrac{1}{2}m_1 - m_2}{m_1 + m_2}\, g = \dfrac{.135 - .075}{.27 + .075}\, 9.8$

$\boxed{a = 1.7\ m/s^2}$

a is positive, which we chose as down the ramp.
Block slides down

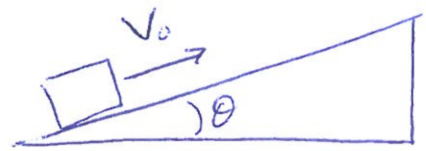
use a to find T:

$T = (.075\,kg)(1.7 + 9.8) = \boxed{0.86\,N}$

Find time using $x = x_0 + v_0 t + \tfrac{1}{2}at^2$
where $x - x_0 = 1\,m$ and $v_0 = 0$ (from rest)

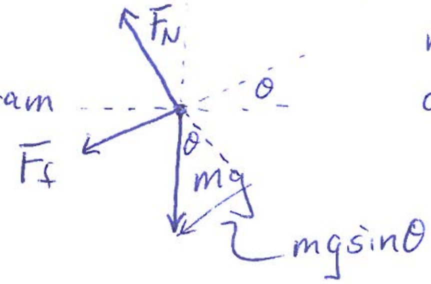
$1\,m = (1.7\tfrac{m}{s^2})\tfrac{1}{2}t^2 \Rightarrow t = \sqrt{\tfrac{2}{1.7}} = \boxed{1.1\,s}$

5-47) A 150-g block is projected up a ramp with an initial speed of 7.0 m/s. The coefficient of kinetic friction between the ramp and the block is 0.23. (*a*) If the ramp is inclined 25° with the horizontal, how far along the surface of the ramp does the block slide before coming to a stop? (*b*) The block then slides back down the ramp. What is the minimum coefficient of static friction between the block and the ramp if the block is not to slide back down the ramp? [$\Delta x = 4.0\text{m}$, $\mu_s = 0.47$]



First find the acceleration (negative) of the box using Newton's second law, then use $2a\Delta x = V_f^2 - V_o^2$ to get distance up the ramp.

Free Body Diagram



normal to ramp: $F_N - mg\cos\theta = \emptyset$

along ramp: $F_f + mg\sin\theta = ma$

$F_N = mg\cos\theta$

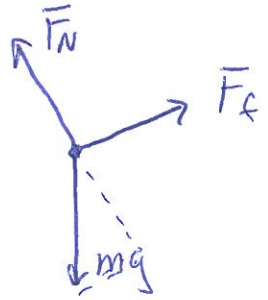
$F_f = \mu_k mg\cos\theta \rightarrow m\mu_k g\cos\theta + mg\sin\theta = ma$

$a = g(\sin\theta + \mu_k\cos\theta)$

direction of a is opposite to velocity, so treat a as $(-)$

$$\Delta x = \frac{V_f^2 - V_o^2}{2a} = \frac{-(7\,m/s)^2}{-2g(\sin\theta + \mu_k\cos\theta)} = \boxed{4.0\ m}$$

If block does not slide back down, $a = 0$ and forces must balance:

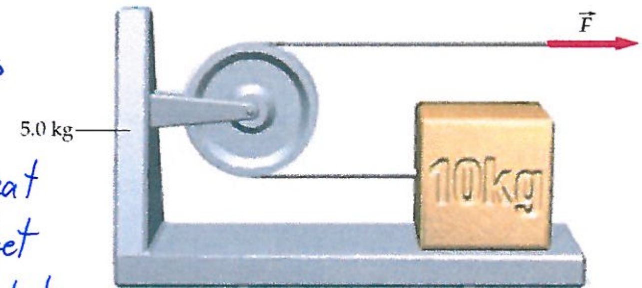


normal to ramp: $F_N = mg\cos\theta$

along ramp $F_f = \mu_k mg\cos\theta = mg\sin\theta$

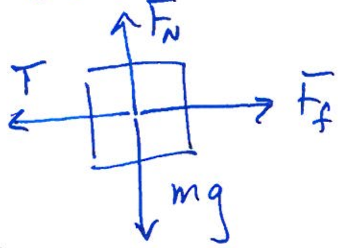
$$\mu_k = \frac{\sin\theta}{\cos\theta} = \boxed{0.47}$$

5-60) A 10.0-kg block rests on a 5.0-kg bracket, as shown below. The 5.0-kg bracket sits on a frictionless surface. The coefficients of friction between the 10.0-kg block and the bracket on which it rests are $\mu_s = 0.40$ and $\mu_k = 0.30$. (*a*) What is the maximum force *F* that can be applied if the 10.0-kg block is not to slide on the bracket? (*b*) What is the corresponding acceleration of the 5.0-kg bracket?
[$F_{max} = 24N$, $a = 1.6 m/s^2$]



If the block does not slide on the bracket, we can treat the block and bracket as one object acted on by the pulling force, F, to get $F = (m_5 + m_{10})\, a$

The block is accelerating to the right with $a = \frac{F}{m_5 + m_{10}}$.
The F.B.D. of the block is



The tension on the rope must be the same everywhere along the rope, so $T = F$.

vertically: $F_N - mg = 0 \Rightarrow F_N = m_{10} g$

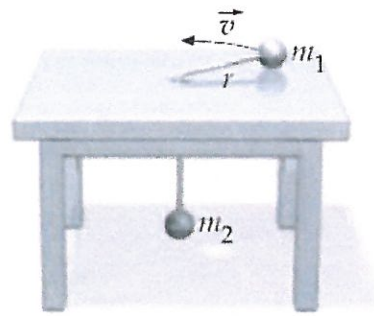
horizontally: $F_f - T = m_{10} a \Rightarrow F_f = \mu_s F_N = F + m_{10} a = F + \frac{m_{10}}{m_5 + m_{10}} F$

$\mu_s m_{10} g = F\left(1 + \frac{m_{10}}{m_5 + m_{10}}\right) \Rightarrow F = \mu_s m_{10} g \frac{(m_5 + m_{10})}{(m_5 + 2m_{10})}$

$F = (0.4)(10\text{ kg})(9.8\, \text{m/s}^2)\left(\frac{15\text{ kg}}{25\text{ kg}}\right) = \boxed{23.5\text{ N}}$

$a = \frac{F}{m_5 + m_{10}} = \mu_s g \frac{m_{10}}{m_5 + 2m_{10}} = (0.4)(9.8\, \text{m/s}^2)\frac{10}{25} = \boxed{1.6\, \text{m/s}^2}$

5-80) An small object of mass m_1 moves in a circular path of radius r on a frictionless horizontal tabletop (below). It is attached to a string that passes through a small frictionless hole in the center of the table. A second object with a mass of m_2 is attached to the other end of the string. Derive an expression for r in terms of m_1, m_2, and the time T for one revolution.



Mass m_1 is moving in a circle on the table, so it must have an acceleration of v^2/r.

Since the only horizontal force acting on m_1 is the tension on the string,

$$T = m_1 a = \frac{mv^2}{r}$$

This same string is holding up m_2, so balancing vertical forces on m_2 (no motion of m_2):

$$T = m_2 g$$

Combining these we get

$$m_2 g = \frac{m_1 v^2}{r} = r = \frac{m_1 v^2}{m_2 g}$$

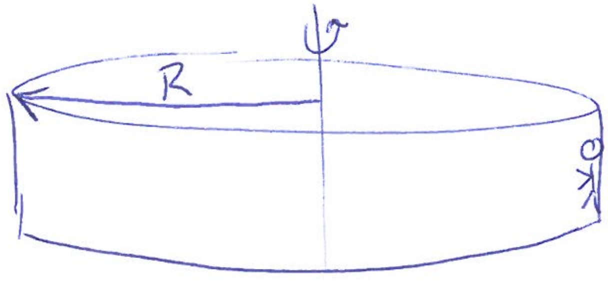
We can compute the speed of m_1 in a circle from

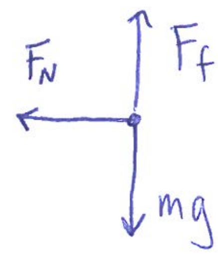
$$v = \frac{2\pi r}{P} \Rightarrow r = \frac{m_1}{m_2 g} \frac{4\pi^2 r^2}{P^2}$$

solving for r:

$$\boxed{r = \frac{m_2 g P^2}{m_1 4\pi^2}}$$

5-131) You are on an amusement park ride with your back against the wall of a spinning vertical cylinder. The floor falls away and you are held up by static friction. Assume your mass is 75 kg. (*a*) Draw a free-body diagram of yourself. (*b*) Use this diagram with Newton's laws to determine the force of friction on you. (*c*) If the radius of the cylinder is 4.0 m and the coefficient of static friction between you and the wall is 0.55. What is the minimum number of revolutions per minute necessary to prevent you from sliding down the wall? Does this answer hold only for you? Will other, more massive, patrons fall downward? Explain. [F_f = 740N, 20 rev/min]





(a)

(b) If there is no vertical motion, $\sum F_y = 0 \Rightarrow F_f = mg$

Force of friction $= (75\text{ kg})(9.8\text{ m/s}^2) = \underline{735\text{ N}}$

(c) If you are moving in a circle, your acceleration is v^2/R, directed toward the center. This acceleration is produced by the normal force, so $F_N = \frac{mv^2}{R}$.

Since $F_f = \mu_s F_N$, we need $735\text{ N} = \mu_s \frac{mv^2}{R}$

To relate speed to revolutions per minute,

$$v = N\,\frac{\text{rev}}{\text{min}} \times 2\pi R\,\frac{\text{m}}{\text{rev}} \times \frac{1\text{ min}}{60\text{ sec}} = NR\frac{\pi}{30}\,\frac{\text{m}}{\text{s}}$$

Plugging this into $mg = \mu_s m \frac{v^2}{R} = \mu_s m \frac{N^2 R^2 \pi^2}{900 R}$

This answer does <u>not</u> depend on my mass

$$\frac{900 \times g}{\mu_s \pi^2 R} = N^2 = 407$$

$$\boxed{N = 20.1\ \frac{\text{rev}}{\text{min}}}$$