PY 205H – Homework #4

8-68) A heavy wooden block rests on a flat table and a high-speed bullet is fired horizontally into the block, the bullet stopping in it. How far will the block slide before coming to a stop? The mass of the bullet is 10.5 g, the mass of the block is 10.5 kg the bullet's impact speed is 750 m/s, and the coefficient of kinetic friction between the block and the table is 0.220. (Assume that the bullet does not cause the block to spin.) [13.0 cm]

We can treat this as two separate events

1) bullet colliding with block → use conservation of mom.
2) block (and bullet) sliding with friction → conservation of energy, including work by friction

Stage 1 initial momentum $p_i = m_{bul} V_o =$

$= (.0105 \text{ kg})(750 \text{ m/s}) =$

final momentum $P_f = (m_{bul} + m_{blo}) V_f$

setting $P_i = P_f \Rightarrow V_f = \dfrac{m_{bul}}{m_{bul} + m_{blo}} V_o$

$= \dfrac{0.0105 \text{ kg}}{10.5105 \text{ kg}} 750 \text{ m/s} = 0.75 \text{ m/s}$

Stage 2 block starts out with $V = 0.75$ m/s and kinetic energy $K = \frac{1}{2}(10.5105 \text{ kg})(0.75 \text{ m/s})^2$

the block will come to a stop when the work done by friction is equal to this original kinetic energy:

$$K_i = W_f$$

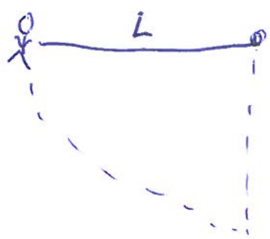
$$\frac{1}{2}(m_{bul} + m_{blo}) V_f^2 = \mu_k (m_{bul} + m_{blo}) g X$$

$$X = \frac{V_f^2}{2g\mu_k} = \frac{(0.75 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)(0.22)} = \underline{.13 \text{ m}}$$

8-70) Tarzan is in the path of a pack of stampeding elephants when Jane swings in to the rescue on a rope vine, hauling him off to safety. The length of the vine is 25m, and Jane starts her swing with the rope horizontal. If Jane's mass is 54 kg, and Tarzan's mass is 82 kg, to what height above the ground will the pair swing after she rescues him? (Assume the rope is vertical when she grabs him.) [3.9 m]

Jane grabbing Tarzan is ~~a~~ perfectly inelastic collision. We can use conservation of energy to find Janes speed just before collision, and then again to find how high they go after the collision.

Stage 1 Jane swinging



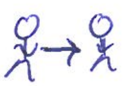
initial $E_{mech} = U_g + K = m_J g L + \emptyset$

final $E_{mech} = U_g + K = \emptyset + \frac{1}{2} m_J V_J^2$

$E_i = E_f \Rightarrow gL = \frac{1}{2} V_J^2 \quad V_J = (2gL)^{1/2}$

Stage 2 Jane collides with Tarzan

Conserve linear momentum

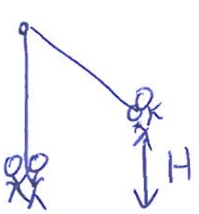


initial $P_x = m_J V_J$

final $P_x = (m_J + m_T) V_{TJ}$

$P_i = P_f \Rightarrow V_{TJ} = \frac{m_J}{m_J + m_T} V_J = \frac{m_J}{m_J + m_T} (2gL)^{1/2}$

Stage 3 Swing back up to height H



initial $E_{mech} = U_g + K = \emptyset + \frac{1}{2}(m_J + m_T)\left[\frac{m_J}{m_J + m_T}\sqrt{2gL}\right]^2$

final $E_{mech} = U_g + K = (m_J + m_T) g H + \emptyset$

$E_i = E_f \Rightarrow (m_J + m_T) g H = \frac{1}{2}(m_J + m_T)\left(\frac{m_J}{m_J + m_T}\right)^2 2gL$

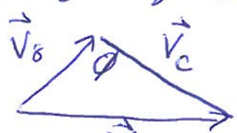
$H = \left(\frac{m_J}{m_J + m_T}\right)^2 L = \left(\frac{54}{136}\right)^2 25m = \underline{3.9 m}$

8-85) In a pool game, the cue ball, which has an initial speed of 5.0 m/s, makes an elastic collision with the eight ball, which is initially at rest. After the collision, the eight ball moves at an angle of 30° to the right of the original direction of the cue ball. Assume that the balls have equal mass. (*a*) Find the direction of motion of the cue ball immediately after the collision. (*b*) Find the speed of each ball immediately after the collision. [(*a*) 60°, (*b*) v_{cb} = 2.50 m/s, v_8 = 4.33 m/s]

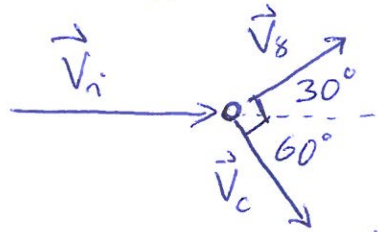
An elastic collision conserves both momentum and energy: $m\vec{V}_i = m\vec{V}_8 + m\vec{V}_c$ where $\vec{V}_i$ is initial velocity of cue ball

$$\tfrac{1}{2}mV_i^2 = \tfrac{1}{2}mV_8^2 + \tfrac{1}{2}mV_c^2$$

$\vec{V}_i = \vec{V}_8 + \vec{V}_c \Rightarrow$



but if $V_i^2 = V_8^2 + V_c^2$, then ϕ must be 90°



and angle of cue ball with respect to $\vec{V}_i$ must be 60°

Now use conservation of x and y momentum

initially $P_y = 0$, so $P_{y,8} + P_{y,c} = 0$

$$V_8 \sin 30° - V_c \sin 60° = 0$$

$$\tfrac{1}{2}V_8 = \tfrac{\sqrt{3}}{2}V_c \Rightarrow \underline{V_c = \tfrac{1}{\sqrt{3}}V_8}$$

initial $P_x = mV_i$, so $P_{x,8} + P_{x,c} = mV_i$

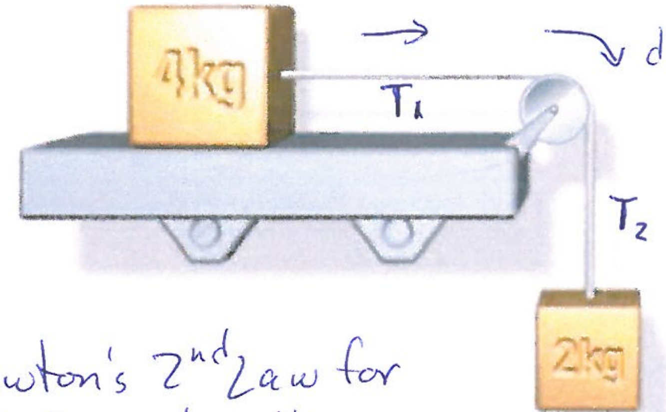
$$V_8 \cos 30° + V_c \cos 60° = V_i$$

$$V_8 \tfrac{\sqrt{3}}{2} + V_c \tfrac{1}{2} = 5 \text{ m/s}$$

$$V_8 \tfrac{\sqrt{3}}{2} + V_8 \tfrac{1}{2\sqrt{3}} = 5 \qquad V_8 = \frac{10}{\sqrt{3} + 1/\sqrt{3}} = \underline{4.33}$$

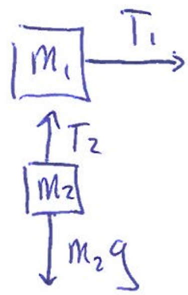
$$V_c = \frac{10}{3+1} = \underline{2.5 \text{ m/s}}$$

9-71) The system shown below consists of a 4.0-kg block resting on a frictionless horizontal ledge. This block is attached to a string that passes over a pulley, and the other end of the string is attached to a hanging 2.0-kg block. The pulley is a uniform disk of radius 8.0 cm and mass 0.60 kg. Find the acceleration of each block and the tensions in the two segments of the string. [3.1 m/s², T_1 = 12 N, T_2 = 13 N]



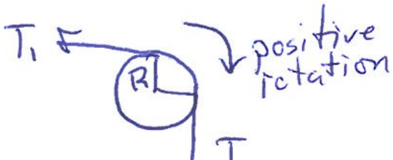
direction of positive motion

Solve Newton's 2^{nd} Law for both blocks and pulley



$$T_1 = m_1 a = (4\,\text{kg})(3.1\,\text{m/s}^2) = \underline{12.4\ \text{N}}$$

$$m_2 g - T_2 = m_2 a \Rightarrow T_2 = m_2(g-a)$$
$$= (2\,\text{kg})(9.8-3.1) = \underline{13.4\ \text{N}}$$



$$\sum \tau = I\alpha$$
$$RT_2 - RT_1 = I\alpha$$
$$R(T_2 - T_1) = Ia/R$$

$\alpha = a/R$, where a is acceleration of edge of pulley, also equal to acceleration of rope if no slipping.

$I = \frac{1}{2}MR^2$ for uniform disk

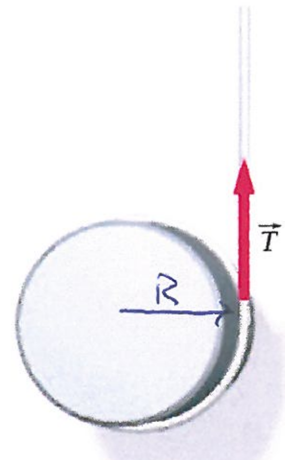
$$T_2 - T_1 = \frac{1}{2}MR^2 \frac{a}{R^2}$$
$$T_2 - T_1 = \frac{1}{2}Ma$$

plugging in T_1 and T_2 from $F = ma$

$$m_2(g-a) - m_1 a = \frac{1}{2}Ma$$
$$m_2 g = \left(\frac{1}{2}M + m_1 + m_2\right)a$$

$$a = \frac{m_2}{\frac{1}{2}M + m_1 + m_2}\, g = \frac{2}{\frac{1}{2}\times 0.6 + 2 + 4}\, 9.8\,\frac{m}{s^2} = \underline{3.1\,m/s^2}$$

9-80) The upper end of the string wrapped around the cylinder in the Figure below is held by a hand that is accelerated upward so that the center of mass of the cylinder does not move as the cylinder spins up. Find (a) the tension in the string, (b) the angular acceleration of the cylinder, and (c) the acceleration of the hand.



$\sum \vec{F} = m\vec{a}$

No linear motion, $\vec{a} = 0$

$\uparrow T$, $\downarrow mg$

$mg - T = 0$

$\boxed{T = mg}$

$\sum \tau = I\alpha$

mg acts at the center of mass, so it does not produce a torque about the center of mass.

Torque due to tension: $\tau = RT$

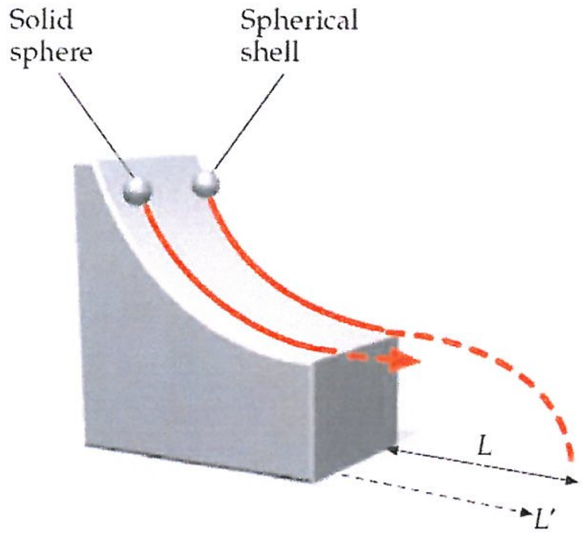
$RT = I\alpha$ where $I = \frac{1}{2}mR^2$ for a uniform cylinder

$$\alpha = \frac{RT}{I} = \frac{2RT}{mR^2} = \boxed{\frac{2T}{mR} = \alpha}$$

The acceleration of the string must match the acceleration of the circumference of the cylinder: $a = \alpha R$, so a of string (and hand) must be

$$a = \alpha R = \frac{2T}{mR}R = \boxed{\frac{2T}{m} = a}$$

9-92) Released from rest at the same height, a thin spherical shell and solid sphere of the same mass m and radius R roll without slipping down an incline through the same vertical drop H. Each is moving horizontally as it leaves the ramp. The spherical shell hits the ground a horizontal distance L from the end of the ramp and the solid sphere hits the ground a distance l' from the end of the ramp. Find the ratio L'/L. [1.09]



Working backwards, if we know the horizontal velocities at the end of the ramp, the distance travelled is

$$L = v\Delta t \quad \text{and} \quad L' = v'\Delta t$$

where Δt is the time for both to fall vertically from the bottom of the ramp to the ground.

Then $\frac{L}{L'} = \frac{V}{V'}$

We can~~d~~ find v, v' using conservation of energy

initial $E_{mech} = mgH$

final $E_{mech} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ where $\omega = v/r$ for rolling

$I = \frac{2}{3}mR^2$ for spherical shell

$I' = \frac{2}{5}mR^2$ for solid sphere

$mgH = \frac{1}{2}mv^2 + \frac{1}{2}\frac{2}{3}mR^2 v^2/R^2$ $\qquad gH = \left(\frac{1}{2}+\frac{1}{3}\right)v^2$

$mgH = \frac{1}{2}mv'^2 + \frac{1}{2}\frac{2}{5}mR^2 v'^2/R^2$ $\qquad gH = \left(\frac{1}{2}+\frac{1}{5}\right)v'^2$

$$\left(\frac{V}{V'}\right)^2 = \frac{\frac{1}{2}+\frac{1}{5}}{\frac{1}{2}+\frac{1}{3}} = \frac{6/10}{5/6} = \frac{36}{50}$$

$$\frac{L'}{L} = \frac{V'}{V} = \sqrt{\frac{50}{36}} = \underline{\underline{1.09}}$$