A Tale of Two Effective Field Theories

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CALCULATION OF SPIN-DEPENDENT PARAMETERS
IN THE LANDAU-MIGDAL THEORY OF NUCLEI

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Abstract: Contributions to the spin-dependent parameter $G'_0$, the coefficient of $\sigma_1 \cdot \sigma_2 \cdot \tau_1 \cdot \tau_2 \cdot \delta(r_1 - r_2)$ in the Fermi-liquid interaction, and to the tensor invariants, are related back to elementary-particle exchange. Once finite-range pion-nucleon interactions are used, almost all of $G'_0$ comes from the $\rho$-exchange nucleon-nucleon potential. Using modern parameterizations of the strength in the $\rho$-channel, we find $G'_0$ to be in the region of 1.5 to 2.4 which agrees well with an empirical determination.

1. Introduction

In the sixties a model for nuclei, based on Landau's theory of normal Fermi liquids, was proposed by Migdal. In this theory a set of Fermi-liquid parameters, describing the particle-hole interaction, is assigned to nuclei heavy enough to develop a central region of saturated matter. In so far as the central density of these nuclei is the same, one set of parameters would describe all nuclei. Assuming spin-isospin isotropy the particle-hole interaction in symmetric nuclear matter is given by

$$\mathcal{F}(k_1, k_2) = F(k_1, k_2) + F'(k_1, k_2) \tau_1 \cdot \tau_2 + G(k_1, k_2) \sigma_1 \cdot \sigma_2 + G'(k_1, k_2) \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2,$$

(1.1)
Motivation

There is a successful effective theory of fermionic many body systems

**Landau Fermi-Liquid Theory**


Predicts collective modes, thermodynamics, transport, . . .

Gauge Theories: Unscreened long range forces

Does a quasi-particle EFT exist?
Effective Field Theories

\[ p = p_F \]

- QCD
- HDET
- NonFL-EFT
- CFLChTh

\[ p = p_F \]
High Density Effective Theory

QCD lagrangian

\[ \mathcal{L} = \bar{\psi} (i \not{D} + \mu \gamma_0 - m) \psi - \frac{1}{4} G^{\alpha}_{\mu \nu} G^{\alpha}_{\mu \nu} \]

Quasi-particles (holes)

\[ E_{\pm} = -\mu \pm \sqrt{\vec{p}^2 + m^2} \approx -\mu \pm |\vec{p}| \]

Effective field theory on \( \nu \)-patches

\[ \psi_{\nu \pm} = e^{-i \mu \nu \cdot x} \left( \frac{1 \pm \vec{\alpha} \cdot \vec{\nu}}{2} \right) \psi \]
High Density Effective Theory, cont

Effective lagrangian for $\psi_{v+}$

$$\mathcal{L} = \sum_v \psi_v^\dagger \left( i\nu \cdot D - \frac{D_\perp^2}{2\mu} \right) \psi_v - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \ldots$$
Four Fermion Operators

quark-quark scattering

$$(v_1, v_2) \rightarrow (v_3, v_4)$$

BCS

Landau

$$\mathcal{L}_{BCS} = \frac{1}{\mu^2} \sum V_l^{\Gamma \Gamma'} R_l^{\Gamma \Gamma'} (\vec{v} \cdot \vec{v'}) \left( \psi_v \Gamma \psi_{-v} \right) \left( \psi_v^{\dagger} \Gamma' \psi_{-v'}^{\dagger} \right),$$

$$\mathcal{L}_{FL} = \frac{1}{\mu^2} \sum F_l^{\Gamma \Gamma'} (\phi) R_l^{\Gamma \Gamma'} (\vec{v} \cdot \vec{v'}) \left( \psi_v \Gamma \psi_{v'} \right) \left( \psi_v^{\dagger} \Gamma' \psi_{v'}^{\dagger} \right)$$
Four Fermion Operators: Matching

Match scattering amplitudes on Fermi surface: forward scattering

\[ \begin{align*}
    &v' \quad v' \\
    &v \quad v
    \\
    +
    \\
    &v' \quad v' \\
    &v \quad v
    \\
    =
    \\
    &v' \quad v' \\
    &v \quad v \quad v' \quad v' \\
    +
    \\
    &v' \quad v' \\
    &v \quad v
\\
\end{align*} \]

Color-flavor-spin symmetric terms

\[ f_0^s = \frac{C_F}{4N_cN_f} \frac{g^2}{p_F^2}, \quad f_i^s = 0 \quad (i > 1) \]
Power Counting

Naive power counting

\[ \mathcal{L} = \hat{\mathcal{L}} \left( \psi, \psi^\dagger, \frac{D_{||}}{\mu}, \frac{D_{\perp}}{\mu}, \frac{\bar{D}_{||}}{\mu}, \frac{m}{\mu} \right) \]

Problem: hard loops (large $N_{\bar{\nu}}$ graphs)

\[ \sum_{\bar{\nu}} \int \frac{d^2 \ell_{\perp}}{(2\pi)^2} = \frac{\mu^2}{2\pi^2} \int \frac{d\Omega}{4\pi}. \]

Have to sum large $N_{\bar{\nu}}$ graphs
Effective Theory for $l < m$

$$\mathcal{L} = \psi_v^\dagger \left( i v \cdot D - \frac{D^2}{2\mu} \right) \psi_v + \mathcal{L}_{4f} - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \mathcal{L}_{HDL}$$

$$\mathcal{L}_{HDL} = -\frac{m^2}{2} \sum_v G^a_{\mu\alpha} \frac{v^\alpha v^\beta}{(v \cdot D)^2} G^b_{\mu\beta}$$

Transverse gauge boson propagator

$$D_{ij}(k) = \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{k_0^2 - \vec{k}^2 + i \frac{\pi}{2} m^2 \frac{k_0}{|\vec{k}|}} ,$$

Scaling of gluon momenta

$$|\vec{k}| \sim k_0^{1/3} m^{2/3} \gg k_0 \quad \text{gluons are very spacelike}$$
Non-Fermi Liquid Effective Theory

Gluons very spacelike $|\vec{k}| \gg |k_0|$. Quark kinematics?

$$k_0 \sim k_\parallel + \frac{k_\perp^2}{2\mu}$$

Scaling relations

$$k_\perp \sim m^{2/3} k_0^{1/3}, \quad k_\parallel \sim m^{4/3} k_0^{2/3} / \mu$$

Propagators

$$S_{\alpha\beta} = \frac{-i\delta_{\alpha\beta}}{p_\parallel + \frac{p_\perp^2}{2\mu} - i\epsilon \text{sgn}(p_0)}$$

$$D_{ij} = \frac{-i\delta_{ij}}{k_\perp^2 - i\frac{\pi}{2} m^2 \frac{k_0}{k_\perp}}.$$
Non-Fermi Liquid Expansion

Scale momenta \((k_0, k_||, k_\perp) \rightarrow (sk_0, s^{2/3}k_||, s^{1/3}k_\perp)\)

\[ [\psi] = 5/6 \quad [A_i] = 5/6 \quad [S] = [D] = 0 \]

Scaling behavior of vertices

Systematic expansion in \(\epsilon^{1/3} \equiv (\omega/m)^{1/3}\)
Loop Corrections: Quark Self Energy

\[ g^2 C_F \int \frac{dk_0}{2\pi} \int \frac{dk_\perp^2}{(2\pi)^2} \frac{k_\perp}{k_\perp^3 + i\eta k_0} \]
\[ \times \int \frac{dk_\parallel}{2\pi} \frac{\Theta(p_0 + k_0)}{k_\parallel + p_\parallel - \frac{(k_\perp + p_\perp)^2}{2\mu} + i\epsilon} \]

Transverse momentum integral logarithmic

\[ \int \frac{dk_\perp^3}{k_\perp^3 + i\eta k_0} \sim \log \left( \frac{\Lambda}{k_0} \right) \]

Quark self energy

\[ \Sigma(p) = \frac{g^2}{9\pi^2} p_0 \log \left( \frac{\Lambda}{|p_0|} \right) \]
Quark Self Energy, cont

Higher order corrections?

\[
\Sigma(p) = \frac{g^2}{9\pi^2} \left( p_0 \log \left( \frac{2^{5/2} m}{\pi |p_0|} \right) + i \frac{\pi}{2} p_0 \right) + O \left( \epsilon^{5/3} \right)
\]

Scale determined by electric gluon exchange

No \( p_0 [\alpha_s \log(p_0)]^n \) terms

quasi-particle velocity vanishes as

\[
v \sim \log(\Lambda/\omega)^{-1}
\]

anomalous term in the specific heat

\[
c_v \sim \gamma T \log(T)
\]
Vertex Corrections, Migdal’s Theorem

Corrections to quark gluon vertex

\[ \sim gv(1 + O(\epsilon^{1/3})) \]

Analogous to electron-phonon coupling

Can this fail? Yes, if external momenta fail to satisfy \( p_\perp \gg p_0 \)

\[ \sim e g^2 \frac{v_\mu}{9\pi^2} \log(\epsilon) \]
Superconductivity

Same phenomenon occurs in anomalous self energy

\[
\frac{g^2}{18\pi^2} \int dq_0 \log \left( \frac{\Lambda_{BCS}}{|p_0 - q_0|} \right) \frac{\Delta(q_0)}{\sqrt{q_0^2 + \Delta(q_0)^2}}
\]

\[\Lambda_{BCS} = 256\pi^4 g^{-5} \mu\] determined by electric exchanges

Have to sum all planar diagrams, non-planar suppressed by \(\epsilon^{1/3}\)

Solution at next-to-leading order (includes normal self energy)

\[\Delta_0 = 2\Lambda_{BCS} \exp \left( -\frac{\pi^2 + 4}{8} \right) \exp \left( -\frac{3\pi^2}{\sqrt{2}g} \right) \Delta_0 \sim 50\text{ MeV}\]


Summary

Systematic low energy expansion in \((\omega/m)^{1/3}\) and \(\log(\omega/m)\)

Standard FL channels (BCS, ZS, ZS'): Ladder diagrams have to be summed, kernel has perturbative expansion
Pion condensation and density isomerism in nuclear matter

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We follow the treatment of the σ model in the mean-field approximation and make the ansatz

\[ \langle \sigma \rangle = f_\sigma \cos \theta, \quad \langle \pi^\pm \rangle = f_\pi \sin \theta e^{i \varphi}, \quad \langle \pi^0 \rangle = 0 \]  

(1)

for the meson fields, the chiral invariant being

\[ f_\sigma = \sigma^2 + \mathbf{r} \cdot \mathbf{v} \quad \text{with} \quad f_\sigma = 94.5 \text{ MeV}, \quad \text{the pion decay constant,} \quad k \text{ is the pion momentum.} \]

This ansatz results in a liquid condensate (the nuclear matter phase) see Dutry's article.\textsuperscript{17} With the ansatz (1) we get the total Hamilton density

\[ H = \mathcal{H}_{\text{M}} + \mathcal{H}_{\text{N*N}} \]

(2a)

where the meson part is given by

\[ H_{\text{M}} = \frac{1}{2} f_\sigma^2 r^2 \sin^2 \theta + f_\pi^2 m_\pi^2 (1 - \cos \theta) \]

(2b)

where \( f_\pi^2 m_\pi^2 \) is added in order to set \( H_{\text{M}} = 0 \) for \( \theta = 0 \). The nucleon and interaction part \( \mathcal{H}_{\text{N*N}} \) is (in nonrelativistic approximation) given by

\[ \mathcal{H}_{\text{N*N}} = \mathcal{H}_{\text{N}} \left( \frac{15 k^4}{2 M^2} - \frac{k^4}{2 M} \right) \]

(3)

where \( g_A = f_\pi M^2 / \sqrt{4 \pi} \) and \( g_A = 6.7 m_\pi \). Diagonalization of \( \mathcal{H}_{\text{N*N}} \) in isospin space gives the quasiparticle energies

\[ E_\mathbf{p} = \frac{p^2}{2 M} + \frac{k^2 \cos \theta}{8 M} - a \left( a^2 + b^2 \right)^{1/2} \]

(4a)

where

\[ a = \frac{-\mathbf{p} \cdot \mathbf{k}}{2 M} \cos \theta, \quad b = \frac{k}{2 g_A} \sin \theta. \]

(4b)

The ground state energy density of the system is then obtained by minimization of

\[ \mathcal{E} = \mathcal{H}_{\text{M}} + 2 \sum \int \frac{d^3 y}{(2\pi)^3} E_\mathbf{p} \Theta [\lambda - E_\mathbf{p}] \]

(5)
CFL Phase

Consider $N_f = 3$ ($m_i = 0$)

\[
\langle q_i^a q_j^b \rangle = \phi \epsilon^{abI} \epsilon_{ijI}
\]

\[
\langle ud \rangle = \langle us \rangle = \langle ds \rangle
\]

\[
\langle rb \rangle = \langle rg \rangle = \langle bg \rangle
\]

Symmetry breaking pattern:

\[
SU(3)_L \times SU(3)_R \times [SU(3)]_C
\]

\[
\times U(1) \to SU(3)_C + F
\]

All quarks and gluons acquire a gap

\[
\langle \psi_L \psi_L \rangle = -\langle \psi_R \psi_R \rangle
\]
EFT in the CFL Phase

Consider HDET with a CFL gap term

\[ \mathcal{L} = \text{Tr} \left( \psi_L^\dagger (iv \cdot D) \psi_L \right) + \frac{\Delta}{2} \left\{ \text{Tr} \left( X^\dagger \psi_L X^\dagger \psi_L \right) - \kappa \left[ \text{Tr} \left( X^\dagger \psi_L \right) \right]^2 \right\} + (L \leftrightarrow R, X \leftrightarrow Y) \]

\[ \psi_L \to L \psi_L C^T, \quad X \to L X C^T, \quad \langle X \rangle = \langle Y \rangle = \mathbb{1} \]

Quark loops generate a kinetic term for \( X, Y \)

Integrate out gluons, identify low energy fields \((\xi = \Sigma^{1/2})\)

\[ \Sigma = X Y^\dagger \]

\( [8]+[1] \) GBs

\[ N_L = \xi (\psi_L X^\dagger) \xi^\dagger \]

\( [8]+[1] \) Baryons
Effective theory: (CFL) baryon chiral perturbation theory

\[ \mathcal{L} = \frac{f_{\pi}^2}{4} \left\{ \text{Tr} \left( \nabla_0 \Sigma \nabla_0 \Sigma^\dagger \right) - v_{\pi}^2 \text{Tr} \left( \nabla_i \Sigma \nabla_i \Sigma^\dagger \right) \right\} \\
+ \text{Tr} \left( N^\dagger i\nu^\mu D_\mu N \right) - D \text{Tr} \left( N^\dagger \nu^\mu \gamma_5 \{ A_\mu, N \} \right) \\
- F \text{Tr} \left( N^\dagger \nu^\mu \gamma_5 [ A_\mu, N ] \right) + \frac{\Delta}{2} \left\{ \text{Tr} (NN) - [\text{Tr} (N)]^2 \right\} \]

with \( D_\mu N = \partial_\mu N + i[\mathcal{V}_\mu, N] \)

\[ \mathcal{V}_\mu = -\frac{i}{2} \left( \xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi \right) \]

\[ A_\mu = -\frac{i}{2} \xi \left( \partial_\mu \Sigma^\dagger \right) \xi \]

\[ f_{\pi}^2 = \frac{21 - 8 \log 2}{18} \left( \frac{\mu^2}{2\pi^2} \right) \quad v_{\pi}^2 = \frac{1}{3} \quad D = F = \frac{1}{2} \]
Mass Terms: Match HDET to QCD

\[ \mathcal{L} = \psi_R^\dagger \frac{MM^\dagger}{2\mu} \psi_R + \psi_L^\dagger \frac{M^\dagger M}{2\mu} \psi_L \]

\[ + \frac{C}{\mu^2} (\psi_R^\dagger M \lambda^a \psi_L)(\psi_R^\dagger M \lambda^a \psi_L) \]

mass corrections to FL parameters \( \hat{\mu} \) and \( F^0(++) \rightarrow (--\) \)
Phase Structure and Spectrum

Phase structure determined by effective potential

\[ V(\Sigma) = \frac{f^2}{2} \text{Tr} \left( X_L \Sigma X_R \Sigma^\dagger \right) - A \text{Tr}(M \Sigma^\dagger) - B_1 \left[ \text{Tr}(M \Sigma^\dagger) \right]^2 + \ldots \]

\[ V(\Sigma_0) \equiv \min \]

Fermion spectrum determined by

\[ \mathcal{L} = \text{Tr} \left( N^\dagger i v^\mu D_\mu N \right) + \text{Tr} \left( N^\dagger \gamma_5 \rho_A N \right) + \frac{\Delta}{2} \left\{ \text{Tr} \left( N N \right) - \left[ \text{Tr} \left( N \right) \right]^2 \right\}, \]

\[ \rho_{V,A} = \frac{1}{2} \left\{ \xi \frac{M^\dagger M}{2p_F} \xi^\dagger \pm \xi^\dagger \frac{M M^\dagger}{2p_F} \xi \right\} \quad \xi = \sqrt{\Sigma_0} \]
Phase Structure and Spectrum

meson condensation: CFLK
s-wave condensate
gapless modes? (gCFLK)
p-wave condensation
Instabilities

Consider meson current

\[ \Sigma(x) = U_Y(x) \Sigma_K U_Y(x)^\dagger \quad U_Y(x) = \exp(i\phi_K(x)\lambda_8) \]

\[ \vec{V}(x) = \frac{\vec{\nabla}\phi_K}{4} (-2\hat{I}_3 + 3\hat{Y}) \quad \vec{A}(x) = \vec{\nabla}\phi_K (e^{i\phi_K} \hat{u}^+ + e^{-i\phi_K} \hat{u}^-) \]

Gradient energy

\[ \mathcal{E} = \frac{f^2}{2} v^2 \nu J^2_K \quad \vec{j}_K = \vec{\nabla}\phi_K \]

Fermion spectrum

\[ \omega_l = \Delta + \frac{l^2}{2\Delta} - \frac{4\mu_s}{3} - \frac{1}{4} \vec{v} \cdot \vec{j}_K \]

\[ \mathcal{E} = \frac{\mu^2}{2\pi^2} \int dl \int d\Omega \omega_l \Theta(-\omega_l) \]
Energy Functional

\[ \frac{3\mu_s - 4\Delta}{\Delta} \bigg|_{crit} = ah_{crit} \quad h_{crit} = -0.067 \quad a = \frac{2}{15^2 c^2 \pi v^4_{\pi}} \]

[Figures include baryon current \( j_B = \alpha_B / \alpha_K j_K \)]
Notes

No net current, meson current canceled by backflow of gapless modes

\[
\frac{\delta \mathcal{E}}{\delta \nabla \phi} = 0
\]

Instability related to “chromomagnetic instability”

CFL phase: gluons carry $SU(3)_F$ quantum numbers

Meson current equivalent to a color gauge field
Happy Birthday
Wolfram, Peter & Gerry