Strongly interacting quantum fluids: Experimental status

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Perfect fluids: The contenders

QGP (T=180 MeV)

Trapped Atoms
(T=0.1 neV)

Liquid Helium
(T=0.1 meV)
I. Experiment (liquid helium)

Kapitza (1938)
viscosity vanishes below $T_c$
capillary flow viscometer

Hollis-Hallett (1955)
roton minimum, phonon rise
rotation viscometer

$$\eta/s \simeq 0.8 \hbar/k_B$$
II. Heavy ion collision: Geometry

rapidity: \( y = \frac{1}{2} \log \left( \frac{E + p_z}{E - p_z} \right) \)

transverse momentum: \( p_T^2 = p_x^2 + p_y^2 \)
Bjorken expansion

Experimental observation: At high energy ($\Delta y \to \infty$) rapidity distributions of produced particles (in both pp and AA) are “flat"

$$\frac{dN}{dy} \simeq \text{const}$$

Physics depends on proper time $\tau = \sqrt{t^2 - z^2}$, not on $y$

All comoving ($v = z/t$) observers are equivalent

Analogous to Hubble expansion
Bjorken expansion

$\tau = \text{const}$

$\eta = \text{const}$

QGP

pre-equilibrium

hadrons

projectile

target

z
Bjorken expansion: Hydrodynamics

Boost invariant expansion

\[ u^\mu = \gamma(1, 0, 0, v_z) = (t/\tau, 0, 0, z/\tau) \]

solves Euler equation (no longitudinal acceleration)

\[ \partial^\mu (su_\mu) = 0 \quad \Rightarrow \quad \frac{d}{d\tau} [\tau s(\tau)] = 0 \]

Solution for ideal Bj hydrodynamics

\[ s(\tau) = \frac{s_0 \tau_0}{\tau} \quad \Rightarrow \quad T = \frac{const}{\tau^{1/3}} \]

Exact boost invariance, no transverse expansion, no dissipation, ...
**Numerical estimates**

Total entropy in rapidity interval \([y, y + \Delta y]\)

\[
S = s \pi R^2 z = s \pi R^2 \tau \Delta y = (s_0 \tau_0) \pi R^2 \Delta y
\]

\[
s_0 \tau_0 = \frac{1}{\pi R^2} \frac{S}{\Delta y}
\]

Use \(S/N \simeq 3.6\)

\[
s_0 = \frac{3.6}{\pi R^2 \tau_0} \left( \frac{dN}{dy} \right) \quad \text{Bj estimate}
\]

\[
\epsilon_0 = \frac{1}{\pi R^2 \tau_0} \left( \frac{dE_T}{dy} \right)
\]

Depends on initial time \(\tau_0\)
BNL and RHIC
Multiplicities

Au+Au 19.6 GeV

Au+Au 130 GeV

Au+Au 200 GeV

Phobos White Paper (2005)
Bjorken expansion

![Diagram showing Bjorken expansion with energy density and time axes. The diagram includes markers for threshold for QGP formation, formation time, and possible EOS.](image)
Chemical equilibrium at freezeout

Andronic et al. (2006)
Collective behavior: Radial flow

Radial expansion leads to blue-shifted spectra in Au+Au

\[ \nu_T \sim 0.6c! \]

\[ m_T = \sqrt{p_T^2 + m^2} \]
Collective behavior: Elliptic flow

Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy

\[ p_0 \frac{dN}{d^3p} \bigg|_{p_z=0} = v_0(p_\perp) \left( 1 + 2v_2(p_\perp) \cos(2\phi) + \ldots \right) \]
Elliptic flow II: Multiplicity scaling

Viscous Corrections

Longitudinal expansion: Bj expansion solves Navier-Stokes equation

entropy equation

\[ \frac{1}{s} \frac{ds}{d\tau} = -\frac{1}{\tau} \left( 1 - \frac{4}{3} \frac{\eta + \zeta}{sT\tau} \right) \]

Viscous corrections small if \( \frac{4}{3} \frac{\eta}{s} + \frac{\zeta}{s} \ll (T\tau) \)

early \( T\tau \sim \tau^{2/3} \quad \eta/s \sim \text{const} \quad \eta/s < \tau_0 T_0 \)

late \( T\tau \sim \text{const} \quad \eta \sim T/\sigma \quad \tau^2/\sigma < 1 \)

Hydro valid for \( \tau \in [\tau_0, \tau_{fr}] \)
Viscous corrections to $T_{ij}$ (radial expansion)

$$T_{zz} = P - \frac{4}{3} \frac{\eta}{\tau} \quad T_{xx} = T_{yy} = P + \frac{2}{3} \frac{\eta}{\tau}$$

increases radial flow (central collision)

decreases elliptic flow (peripheral collision)

Modification of distribution function

$$\delta f = \frac{3}{8} \frac{\Gamma_s}{T^2} f_0 (1 + f_0) p_{\alpha} p_{\beta} \nabla \langle \alpha u^\beta \rangle$$

Correction to spectrum grows with $p_{\perp}^2$

$$\frac{\delta (dN)}{dN_0} = \frac{\Gamma_s}{4 \tau_f} \left( \frac{p_{\perp}}{T} \right)^2$$
Elliptic flow III: Viscous effects

Elliptic flow IV: Systematic trends

Deviation from ideal hydro increases for more peripheral events increases with $p_\perp$

source: R. Snellings (STAR)
Elliptic flow V: Predictions for LHC

Romatschke, Luzum (2009)

Busza (QM 2009)
Elliptic flow VI: Recombination

“quark number” scaling of elliptic flow

\[ \text{Anisotropy Parameter } v_2 \]

\[ \text{Hydro model} \]

\[ \text{PHENIX Data} \quad \text{STAR Data} \]

\[ \pi^+ \pi^- \quad h^+ h^- \quad p^+ p^- \quad \Lambda^+ \Lambda^- \]

\[ p_{\text{mes}} = 2p_{\text{qu}} \]

\[ p_{\text{bar}} = 3p_{\text{qu}} \]
Jet quenching

\[ R_{AA} = \frac{n_{AA}}{N_{\text{coll}} n_{pp}} \]

Jet quenching II

Disappearance of away-side jet

Jet quenching III: The Mach cone

azimuthal multiplicity $dN/d\phi$
(high energy trigger particle at $\phi = 0$)

wake of a fast quark in $\mathcal{N} = 4$ plasma

Jet quenching: Theory

energy loss governed by

$$\hat{q} = \rho \int q_\perp^2 dq_\perp^2 \frac{d\sigma}{dq_\perp^2}$$

larger than pQCD predicts? relation to $\eta$? ($\hat{q} \sim 1/\eta$?)

also: large energy loss of heavy quarks

Where are we?

observe almost ideal fluid behavior, initial conditions well above critical energy density.

systematics require $0 < \eta/s < 0.4$; more studies needed, LHC elliptic flow will be very interesting.

jet quenching large; very detailed studies under way. LHC will provide unprecedented range.

heavy flavors: large energy loss seen, flavor studies $(c/b)$ under way.
III. Experiment: Cold gases

transverse expansion   expansion (rotating trap)   collective modes

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \vec{v}) = 0
\]

\[
mn \frac{\partial \vec{v}}{\partial t} + mn \left( \vec{v} \cdot \nabla \right) \vec{v} = -\nabla P - n \nabla V
\]
Scaling flows

Universal equation of state

\[ P = \frac{n^{5/3}}{m} f \left( \frac{mT}{n^{2/3}} \right) \]

Equilibrium density profile (local density approximation)

\[ n_0(x) = n(\mu(x), T) \quad \mu(x) = \mu_0 \left( 1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2} - \frac{z^2}{R_z^2} \right) \]

Scaling Flow: Stretch and rotate profile

\[ \mu_0 \rightarrow \mu_0(t), \quad T \rightarrow T_0(\mu_0(t)/\mu_0), \quad R_x \rightarrow R_x(t), \ldots \]

Linear velocity profile

\[ \vec{v}(x, t) = \frac{1}{2} \nabla \left( \alpha_x x^2 + \alpha_y y^2 + \alpha_z z^2 + \alpha_{xy} \right) + \omega \hat{z} \times \vec{x}. \]

“Hubble flow”
Almost ideal fluid dynamics

Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy

O’Hara et al. (2002)
Almost ideal fluid dynamics

Radial breathing mode

Ideal fluid hydrodynamics \((P \sim n^{5/3})\)

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \vec{v}) = 0
\]

\[
\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = -\frac{\nabla P}{mn} - \frac{\nabla V}{m}
\]

Hydro frequency at unitarity

\[
\omega = \sqrt{\frac{10}{3}} \omega_{\perp}
\]

experiment: Kinast et al. (2005)
Dissipation (scaling flows)

Energy dissipation ($\eta, \zeta, \kappa$: shear, bulk viscosity, heat conductivity)

\[
\dot{E} = -\frac{1}{2} \int d^3 x \, \eta(x) \left( \partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right)^2 \\
- \int d^3 x \, \zeta(x) (\partial_i v_i)^2 - \frac{1}{T} \int d^3 x \, \kappa(x) (\partial_i T)^2
\]

Have $\zeta = 0$ and $T(x) = \text{const}$. Universality implies

\[
\eta(x) = s(x) \alpha_s \left( \frac{T}{\mu(x)} \right)
\]

\[
\int d^3 x \, \eta(x) = S \langle \alpha_s \rangle
\]
Collective modes: Small viscous correction exponentiates

\[ a(t) = a_0 \cos(\omega t) \exp(-\Gamma t) \]

\[ \langle \eta/s \rangle = (3N\lambda)^{1/3} \left( \frac{\Gamma}{\omega_\perp} \right) \left( \frac{E_0}{E_F} \right) \left( \frac{N}{S} \right) \]

Kinast et al. (2006), Schaefer (2007)
Navier-Stokes equation

Option 1: Moment method

\[ \int d^3x \, x_k (\rho \dot{v}_i + \ldots) = \int d^3x \, x_k (-\nabla_i P - \nabla_j \delta \Pi_{ij}) \]

Only involves \( \langle \eta \rangle / E_0 \).

Option 2: Scaling ansatz for \( \eta(\mu, T) \)

\[ \eta(n, T) = \eta_0 (mT)^{3/2} + \eta_1 \frac{P(n, T)}{T} \]

Option 3: Numerical solutions.
Dissipation

\[ R_i \quad [\mu m]\]

\[ R_z \quad R_\perp \]

\[ t [ms] \]

\[ \theta [^\circ] \]

\[ E/E_F = 0.56 \]

\[ E/E_F = 2.1 \]

Dissipation

\[
\frac{(\delta t_0)}{t_0} = \begin{cases} 0.008 \\ 0.024 \end{cases} \left( \frac{\langle \alpha_s \rangle}{1/(4\pi)} \right) \left( \frac{2 \cdot 10^5}{N} \right)^{1/3} \left( \frac{S/N}{2.3} \right) \left( \frac{0.85}{E_0/E_F} \right)
\]

\( t_0 \): “Crossing time” \( (b_\perp = b_z, \theta = 45^\circ) \)

\( a \): amplitude
Time Scales

\[ R_i \]
\[ R_{\perp} \]
\[ R_z \]

\[ t_{\text{acc}} \]
\[ t_{\text{diss}} \]
\[ t_{\text{fr}} \]
\[ t_{\text{cross}} \]
Where are we?

high temperature \((T > 2.5T_c)\) dominated by corona

low temperature \((T \sim T_c)\): evidence for low viscosity \((\eta/s < 0.4)\) core

also seen in “irrotational flow” data

full (2nd order hydro or hydro+kin) analysis needed
Remarkably, the best fluids that have been observed are the coldest and the hottest fluid ever created in the laboratory, cold atomic gases ($10^{-6}$K) and the quark gluon plasma ($10^{12}$K) at RHIC.

Both of these fluids come close to a bound on the shear viscosity that was first proposed based on calculations in string theory, involving non-equilibrium evolution of back holes in 5 (and more) dimensions.