From Trapped Atoms
To Liberated Quarks

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Heavy Ion Collision

Star TPC
Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy.

Source: U. Heinz (2005)
Elliptic Flow II

Requires “perfect” fluidity ($\eta/s < 0.1$ ?)

(s)QGP saturates (conjectured) universal bound $\eta/s = 1/(4\pi)$?
Designer Fluids

Atomic gas with two spin states: “↑” and “↓”

Feshbach resonance

\[ a(B) = a_0 \left( 1 + \frac{\Delta}{B - B_0} \right) \]

“Unitarity” limit \( a \to \infty \)

\[ \sigma = \frac{4\pi}{k^2} \]
Elliptic Flow

Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy.
Perfect Liquids

sQGP \((T=180 \text{ MeV})\)

Trapped Atoms \((T=1 \text{ neV})\)

Neutron Matter \((T=1 \text{ MeV})\)
Universality

What do these systems have in common?
dilute: \( r \rho^{1/3} \ll 1 \)
strongly correlated: \( a \rho^{1/3} \gg 1 \)

Feshbach Resonance in \(^6\text{Li}\)

Neutron Matter
Questions

Equation of State

Critical Temperature

Transport: Shear Viscosity, ...

Stressed Pairing
I. Equation of State
Universal Equation of State

Consider limiting case ("Bertsch" problem)

\[(k_Fa) \to \infty \quad (k_Fr) \to 0\]

Universal equation of state

\[\frac{E}{A} = \xi \left(\frac{E}{A}\right)_0 = \xi \frac{3}{5} \left(\frac{k_F^2}{2M}\right)\]

How to find \(\xi\)?

- Numerical Simulations
- Experiments with trapped fermions
- Analytic Approaches
**Effective Field Theory**

Effective field theory for pointlike, non-relativistic neutrons

\[ \mathcal{L}_{\text{eff}} = \psi^\dagger \left( i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 + \frac{C_2}{16} \left[ (\psi \psi)^\dagger (\psi \nabla^2 \psi) + h.c. \right] + \ldots \]

\[ (a, r, \ldots) \Rightarrow (C_0, C_2, \ldots) \]

Partition Function (Hubbard-Stratonovich field \( s \), Fermion matrix \( Q \))

\[ Z = \int \mathcal{D}s \exp \left[ -S' \right], \quad S' = -\log(\det(Q)) + V(s) \]

\[ C_0 < 0 \text{ (attractive): } \det(Q) \geq 0 \]
Continuum Limit

Fix coupling constant at finite lattice spacing

\[
\frac{M}{4\pi a} = \frac{1}{C_0} + \frac{1}{2} \sum_{\vec{p}} \frac{1}{E_{\vec{p}}}
\]

Take lattice spacing \( b, b_\tau \) to zero

\[
\mu b_\tau \to 0 \quad n^{1/3} b \to 0 \quad n^{1/3} a = \text{const}
\]

Physical density fixed, lattice filling \( \to 0 \)

Consider universal (unitary) limit

\[
n^{1/3} a \to \infty
\]
Lattice Results

Canonical $T = 0$ calculation: $\xi = 0.25(3)$ (D. Lee)

Not extrapolated to zero lattice spacing
Pairing gap ($\Delta$) = 0.9 $E_{FG}$

Odd N: $\xi = 0.4$ (Burovski et al., Bulgac et al.)

Even N: $E = 0.44 N E_{FG}$

Other lattice results: $\xi = 0.4$ (Burovski et al., Bulgac et al.)

Experiment: $\xi = 0.27^{+0.12}_{-0.09}$ [1], 0.51 ± 0.04 [2], 0.74 ± 0.07 [3]

Upper and lower critical dimension

Zero energy bound state for arbitrary $d$

$$\psi''(r) + \frac{d-1}{r}\psi'(r) = 0 \quad (r > r_0)$$

$d=2$: Arbitrarily weak attractive potential has a bound state

$$\xi(d=2) = 1$$

$d=4$: Bound state wave function

$$\psi \sim 1/r^{d-2}.$$ Pairs do not overlap

$$\xi(d=4) = 0$$

Conclude $\xi(d=3) \sim 1/2$?

Try expansion around $d = 4$ or $d = 2$?

Epsilon Expansion

EFT version: Compute scattering amplitude \((d = 4 - \epsilon)\)

\[
T = \frac{1}{\Gamma (1 - \frac{d}{2})} \left( \frac{m}{4\pi} \right)^{-d/2} \left( -p_0 + \frac{\epsilon p}{2} \right)^{1-d/2} \approx \frac{8\pi^2 \epsilon}{m^2} \frac{i}{p_0 + \frac{\epsilon p}{2} + i\delta}
\]

\[
g^2 \equiv \frac{8\pi^2 \epsilon}{m^2} \quad D(p_0, p) = \frac{i}{p_0 + \frac{\epsilon p}{2} + i\delta}
\]

Weakly interacting bosons and fermions
Epsilon Expansion

Effective potential

\[ O(1) \quad O(1) \quad O(\epsilon) \]

\[ \xi = \frac{1}{2} \epsilon^{3/2} + \frac{1}{16} \epsilon^{5/2} \ln \epsilon \\
- 0.0246 \epsilon^{5/2} + \ldots \]

\[ \xi(\epsilon = 1) = 0.475 \]

Problem: Higher order corrections large (\( \sim 100\% \))!

Combine \( d = 4 - \epsilon \) and \( d = 2 + \bar{\epsilon} \) (and Pade)

\[ \xi = (0.3 - 0.35) \]
Quark Gluon Plasma Equation of State (Lattice)

Compilation by F. Karsch (SciDAC)
Holographic Duals at Finite Temperature

Thermal (conformal) field theory $\equiv$ $AdS_5$ black hole

CFT temperature $\Leftrightarrow$ Hawking temperature of black hole

CFT entropy $\Leftrightarrow$ Hawking-Bekenstein entropy $= \text{area of event horizon}$

$$s = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s_0$$

Gubser and Klebanov

Extended to transport properties by Policastro, Son and Starinets

$$\eta = \frac{\pi}{8} N_c^2 T^3$$
II. How Large Can $T_c$ Get?
Critical Temperature: From BCS to BEC

BCS

BEC

\[
T_c^{BCS} = \frac{4 \cdot 2^{1/3} \epsilon^\gamma}{\epsilon^{7/3} \pi} \epsilon_F \exp \left(-\frac{\pi}{|k_F a|}\right)
\]

\[
T_c^{BEC} = 3.31 \left(\frac{n^{2/3}}{m}\right)
\]

\[
T_c(a \to \infty) = 0.28\epsilon_F
\]

\[
T_c = 0.21\epsilon_F + O(a_B n^{1/3})
\]
Lattice results: $T_c/T_F = 0.15$ (UMass)

Kinast et al. (2005)
Quark Matter: Color Superconductivity

Weak coupling result

\[ \frac{T_c}{T_F} = \frac{b e^n}{\pi} \exp \left( -\frac{3\pi^2}{\sqrt{2g}} \right) \]

\[ b = 512\pi^4 g^{-5} \left( 2/N_f \right)^{5/2} e^{-\frac{\pi^2+4}{8}} \]

Maximum \( T_c/T_F = 0.025 \). Strong coupling?

Find \( T_c/T_F \approx 0.2 \)

Note: Transition to \( \chi SB \)
Consider \( N_c = 2 \) QCD?
Importance of $T_c/T_F$: Heavy Ion Collisions at Fair
III. Transport Properties
Collective Modes

Radial breathing mode

Ideal fluid hydrodynamics, equation of state $P \sim n^{5/3}$

\[
\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) = 0
\]

\[
\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} = -\frac{1}{mn} \vec{\nabla} (P + nV)
\]

$\omega = \sqrt{\frac{10}{3}} \omega_{\perp}$
Damping of Collective Excitations

\[ T/T_F = (0.5, 0.33, 0.17) \]

\[ \tau \omega: \text{decay time } \times \text{trap frequency} \]

Kinast et al. (2005)
Viscous Hydrodynamics

Energy dissipated due to viscous effects is

$$\dot{E} = -\frac{\eta}{2} \int d^3 x \left( \partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right)^2 - \zeta \int d^3 x (\partial_i v_i)^2,$$

$$\eta, \zeta: \text{shear, bulk viscosity}$$

Shear viscosity to entropy ratio ($\zeta = 0$)

$$\frac{\eta}{s} \sim c_i \times \frac{\Gamma}{\omega_{\perp}} \times \frac{\mu}{\omega_{\perp}} \times \frac{N}{S}$$

$c_i$ determined by Hydro solution

Bruun, Smith, Gelman et al.

Problems: Scaling with $N$; $T$ dependence below $T_c$
IV. Stressed Pairing
Polarized Fermions: From BEC to BCS

Response of paired state to pair breaking stress (e.g. Zeeman field)

\[ L_{\text{ext}} = \delta \mu \psi^\dagger \sigma_3 \psi \]

BEC limit: Tightly bound bosons, no polarization for \( \delta \mu < \Delta \)

\( \delta \mu > \Delta \): Mixture of Fermi and Bose liquid, no phase separation

BCS limit: No homogeneous mixed phase

\( \delta \mu > \delta \mu_{c1} \): LOFF pairing \( \Delta(x) = e^{iqx} \Delta \)
Inhomogeneous pairing

Onset? Consider EFT for gapless fermions interacting with GB’s

\[ \mathcal{L} = \psi^\dagger \left( i \partial_0 - \epsilon(-i \vec{\partial}) - (\vec{\partial} \varphi) \cdot \frac{\vec{\partial}}{2m} \right) \psi + \frac{f_t^2}{2} \dot{\varphi}^2 - \frac{f^2}{2} (\vec{\partial} \varphi)^2 \]

Free energy of state with non-zero current \( v_s = \partial \varphi / m \)

\[ F(v_s) = \frac{1}{2} n m v_s^2 \]

\[ + \int \frac{d^3 p}{(2\pi)^3} \epsilon_v(p) \Theta(-\epsilon_v(p)) \]

Unstable for BCS-type dispersion relation

\[ x \sim \frac{J}{\Delta} \quad h \sim \frac{\delta \mu - \delta \mu_c}{\Delta} \]
Minimal Phase Diagram

\[ \frac{\delta \mu}{\Delta} \]

\begin{align*}
\text{gapless superfluid} \\
\text{homogeneous superfluid} \\
\text{supercurrent} \\
\text{LOFF}
\end{align*}

\[ \frac{1}{a} \quad \text{\(1/a^*\)} \quad \frac{1}{a} \]

\[ \begin{array}{c}
\varepsilon(p) \\
\Delta \\
p
\end{array} \quad \begin{array}{c}
\varepsilon(p) \\
\Delta \\
p
\end{array} \quad \begin{array}{c}
\varepsilon(p) \\
\Delta \\
p
\end{array} \]

\[ \begin{array}{c}
\Delta \\
\text{homog. superfluid} \\
\text{supercurrent state}
\end{array} \quad \begin{array}{c}
E_i \\
\text{homog. BCS}
\end{array} \quad \begin{array}{c}
E_i \\
gapless BCS \\
\text{LOFF}
\end{array} \]

Son & Stephanov (2005)
Experimental Situation

$\delta E_F = 0.36 E_F$

Zwierlein et al. (MIT group)
Color Superconductivity in QCD: Response to $m_s \neq 0$

QCD with three degenerate flavors: CFL pairing

$$\langle q_i^a q_j^b \rangle = (\delta_i^a \delta_j^b - \delta_j^a \delta_i^b) \phi$$

$$\langle ud \rangle = \langle us \rangle = \langle ds \rangle$$

Pair breaking stress due to $\mu_s = m_s^2/(2p_F) \neq 0$

kinematics + electric neutrality + weak equilibrium
Phase Structure of CFL Quark Matter

How does CFL ($\langle ud \rangle = \langle ds \rangle = \langle su \rangle$) pairing respond to $m_s$?

Excitation energy of fermions

Gapless modes appear at $\mu_s(\text{crit}) \sim 0.75\Delta$

Energy density of superfluid phases

$\mu_s(K - \text{cond}) \sim m_u^{2/3} \Delta^{4/3} / \mu$

$\mu_s(GB - \text{cur}) \sim 0.75\Delta$

Trapped atoms provide interesting model system

equation of state of strongly correlated systems (neutron matter, sQGP)

viscosity of strongly correlated systems (sQGP?)

superfluidity at strong coupling ($T_c/T_F$, response to pair breaking fields, precursor phenomena)