The Phases of QCD

Thomas Schaefer

North Carolina State University
Motivation

Different phases of QCD occur in the universe

**Neutron Stars, Big Bang**

Exploring the phase diagram is important to understanding the phase that we happen to live in

Structure of hadrons is determined by the structure of the vacuum

Need to understand how vacuum can be modified

QCD simplifies in extreme environments

Study QCD matter in a regime where quarks and gluons are the correct degrees of freedom
Plan

1. QCD and Symmetries
2. The High Temperature Phase: Theory
3. Exploring QCD at High Temperature: Experiment
4. QCD at Low Density: Nuclear Matter
5. QCD at High Density: Quark Matter
6. Matter at Finite Density: From the Lab to the Stars
Quantum chromodynamics

Elementary fields:

Quarks

- Color: \( a = 1, \ldots, 3 \)
- Spin: \( \alpha = 1, 2 \)
- Flavor: \( f = u, d, s, c, b, t \)

Gluons

- Color: \( a = 1, \ldots, 8 \)
- Spin: \( \epsilon^{\pm}_{\mu} \)

Dynamics: non-abelian gauge theory

\[
\mathcal{L} = \bar{q}_f (i \mathbb{D} - m_f) q_f - \frac{1}{4} G^{a}_{\mu\nu} G^{a}_{\mu\nu}
\]

\[
G^{a}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu
\]

\[
i \mathbb{D} q = \gamma^\mu \left( i \partial_\mu + g A^a_\mu t^a \right) q
\]
“Seeing” Quarks and Gluons
Running Coupling Constant

\[ \alpha_s(Q) \]

\[ \frac{Q}{[\text{GeV}]} \]

\[ \alpha_s(M_Z) = 0.118 \pm 0.003 \]
Asymptotic Freedom

Study modification of classical field by quantum fluctuations

\[ A_\mu = A^{cl}_\mu + \delta A_\mu \]

\[ \frac{1}{g^2} F^2 \rightarrow \left( \frac{1}{g^2} + c \log \left( \frac{k^2}{\mu^2} \right) \right) F^2 \]

\[ \beta(g) = \frac{g^3}{(4\pi)^2} \left\{ \left[ \frac{1}{3} - 4 \right] N_c + \frac{2}{3} N_f \right\} \]

dielectric $\epsilon > 1$ \hspace{1cm} paramagnetic $\mu > 1$ \hspace{1cm} dielectric $\epsilon > 1$

$\mu \epsilon = 1 \Rightarrow \epsilon < 1$
About Units

QCD Lite* is a parameter free theory

The lagrangian has a coupling constant, $g$, but no scale.

After renormalization $g$ becomes scale dependent

$g$ is traded for a scale parameter $\Lambda$

$\Lambda$ is the only scale, the QCD “standard kilogram”

$\Lambda_{QCD} \sim 200 \text{ MeV} \sim 1 \text{ fm}^{-1}$

*QCD Lite is QCD in the limit $m_q \to 0$, $m_Q \to \infty$
Phases of Gauge Theories

Coulomb  Higgs  Confinement

V(r) \sim \frac{e^2}{r} \quad V(r) \sim \frac{e^{-mr}}{r} \quad V(r) \sim kr

Standard Model: U(1) \times SU(2) \times SU(3)
# Phases of Matter

<table>
<thead>
<tr>
<th>phase</th>
<th>order param</th>
<th>broken symmetry</th>
<th>rigidity phenomenon</th>
<th>Goldstone boson</th>
</tr>
</thead>
<tbody>
<tr>
<td>crystal</td>
<td>$\rho_k$</td>
<td>translations</td>
<td>rigid</td>
<td>phonon</td>
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<tr>
<td>magnet</td>
<td>$\vec{M}$</td>
<td>rotations</td>
<td>magnetization</td>
<td>magnon</td>
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<tr>
<td>superfluid</td>
<td>$\langle \Phi \rangle$</td>
<td>particle number</td>
<td>supercurrent</td>
<td>phonon</td>
</tr>
<tr>
<td>supercond.</td>
<td>$\langle \psi \phi \rangle$</td>
<td>gauge symmetry</td>
<td>supercurrent</td>
<td>none (Higgs)</td>
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</tbody>
</table>
Gauge Symmetry

Local gauge symmetry $U(x) \in SU(3)_c$

\[
\begin{align*}
\psi & \rightarrow U\psi \\
A_\mu & \rightarrow U A_\mu U^\dagger + iU \partial_\mu U^\dagger \\
D_\mu \psi & \rightarrow UD_\mu \psi \quad F_{\mu\nu} & \rightarrow UF_{\mu\nu}U^\dagger
\end{align*}
\]

Gauge “symmetries” cannot be broken

Gauge “symmetries” can be realized in different modes

- **Coulomb**
- **Higgs**
- Confined

- d.o.f: 2 (massless) 3 (massive) 3 (massive)

Distinction between Higgs and confinement phase not always sharp
Chiral Symmetry

Define left and right handed fields

\[
\psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\psi
\]

Fermionic lagrangian, \[M = \text{diag}(m_u, m_d, m_s)\]

\[
\mathcal{L} = \bar{\psi}_L (i \not{\!D}) \psi_L + \bar{\psi}_R (i \not{\!D}) \psi_R
\]

\[+ \bar{\psi}_L M \psi_R + \bar{\psi}_R M \psi_L\]

\[M = 0: \text{Chiral symmetry } (L, R) \in SU(3)_L \times SU(3)_R\]

\[
\psi_L \rightarrow L\psi_L, \quad \psi_R \rightarrow R\psi_R
\]
Chiral Symmetry Breaking

Chiral symmetry implies massless, degenerate fermions

\[ m_N^{(1/2)^+} = 935 \text{ MeV} \quad m_{N^*}(1/2)^- = 1535 \text{ MeV} \]

Chiral symmetry is spontaneously broken

\[ \langle \bar{\psi}_L^f \psi_R^g + \bar{\psi}_L^f \psi_R^g \rangle \approx -(230 \text{ MeV})^3 \delta^{fg} \]

\[ SU(3)_L \times SU(3)_R \rightarrow SU(3)_V \quad (G \rightarrow H) \]

Consequences: dynamical mass generation \( m_Q = 300 \text{ MeV} \gg m_q \)

\[ m_N = 890 \text{ MeV} + 45 \text{ MeV} \quad (\text{QCD, 95\%}) + (\text{Higgs, 5\%}) \]
Goldstone Bosons: Consider broken generator $Q^a_5$

$$[H, Q^a_5] = 0 \quad Q^a_5|0\rangle = |\pi^a\rangle \quad \quad H|\pi^a\rangle = HQ^a_5|0\rangle = Q^a_5H|0\rangle = 0$$

Low energy effective theory for the Goldstone modes

**Step 1:** Parameterize $G/H = \text{pseudoscalar GB’s}$

$$U(x): \quad U \rightarrow LUR^\dagger \quad \quad (L, R) \in SU(3)_L \times SU(3)_R$$

Vacuum $U^{fg} = \delta^{fg}$. Massless fluctuations $(G/H)$

$$U(x) = \exp(i\phi^a \lambda^a / f_\pi) \quad \quad \phi^a = (\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \eta)$$

**Step 2:** Write most general $G$ invariant effective lagrangian

$$\mathcal{L} = \frac{f_\pi^2}{4} \Tr[\partial_\mu U \partial^\mu U^\dagger] + \ldots$$

Non-linear sigma model
Expand lagrangian ($SU(2)$ sector)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^a)^2 + \frac{1}{6f_\pi^2} [(\phi^a \partial_\mu \phi^a)^2 - (\phi^a)^2 (\partial_\mu \phi^b)^2] + O \left( \frac{\partial^4}{f_\pi^4} \right)$$

**Step 3:** Low energy expansion (power counting)

$$T_{\pi\pi} \sim k_{1,2}^2 f_\pi^2 + k_{1,2}^4 f_\pi^4 + \cdots$$

Relation to $f_\pi$: Couple weak gauge fields

$$\partial_\mu U \rightarrow (\partial_\mu + igW_\mu^\pm)U$$

$$\mathcal{L} = gf_\pi W_\mu^\pm \partial^\mu \pi^\mp$$
Quark Masses

Non-zero quark masses: \( \mathcal{L} = \bar{\psi}_L M \psi_R + \bar{\psi}_R M^\dagger \psi_R \)

\[ M \rightarrow LMR^\dagger \quad \text{spurion field } M \]

Chiral lagrangian at leading order in \( M \)

\[ \mathcal{L} = B \text{Tr}[MU] + \text{h.c.} \]

Mass matrix \( M = \text{diag}(m_u, m_d m_s) \). Minimize effective potential

\[ U_{vac} = 1, \quad E_{vac} = -B \text{Tr}[M] \quad \langle \bar{\psi} \psi \rangle = -B \]

Expand around \( U_{vac} \): pion mass

\[ m_\pi^2 f_\pi^2 = (m_u + m_d) \langle \bar{\psi} \psi \rangle \]

Chiral expansion

\[ \mathcal{L} = f_\pi^4 \left( \frac{\partial U}{\Lambda_\chi} \right)^m \left( \frac{m_\pi}{\Lambda_\chi} \right)^n \quad \Lambda_\chi = 4\pi f_\pi \]
Symmetries of the QCD Vacuum: Summary

Local $SU(3)$ gauge symmetry

confined: $V(r) \sim kr$

Chiral $SU(3)_L \times SU(3)_R$ symmetry

spontaneously broken to $SU(3)_V$

Axial $U(1)_A$ symmetry

anomalous: $\partial_\mu A_\mu^0 = \frac{N_f}{16\pi^2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}$

Vectorial $U(1)_B$ symmetry

unbroken: $B = \int d^3x \psi^\dagger \psi$ conserved
Notes

QCD with general $N_f, N_c$ (with or without SUSY)

find theories without confinement and/or chiral symmetry breaking

QCD with $N_f = N_c = 3$

confinement implies chiral symmetry breaking

symmetry breaking pattern $SU(3)_L \times SU(3)_R \to SU(3)_V$ unique*

order parameter $\langle \bar{\psi}\psi \rangle \neq 0$
QCD Phase Diagram: $N_c$ and $N_f$
Approaching the Phase Diagram:
Symmetries and Weak Coupling Arguments
Approaching the Phase Diagram:
Strongly Correlated Phases

\[ \mu \quad e \quad pion \quad QGP \quad \mu \quad pion \quad gas \quad critical \quad sQGP \quad Hagedorn \quad \text{critical} \quad \text{color} \quad \text{superconductor} \]

\[ \text{neutron gas} \quad \text{BEC–BCS} \quad \text{nuclear liquid} \quad \text{critical} \]

22
Approaching the Phase Diagram:

Experiments and Numerical Simulations