QCD at Finite Density

From the Lab to the Stars
Properties of Neutron Stars

mass \( m \simeq (1 - 2)m_\odot \)

radius \( r \simeq (10 - 15) \text{ km} \)

spin period \( \tau \geq 1 \text{ msec} \)

temperature \( T = (1 \text{ keV} \ldots 1 \text{ MeV}) \)

magnetic field \( B \simeq (10^{12} \ldots 10^{16}(?) \text{ Gauss} \)

central density \( \rho \simeq (2.5 - 8)\rho_0 \)
Evolution of Neutron Stars

Lattimer and Prakash, Science (2005)
Composition of Neutron Stars

- hydrogen/heatmosphere
- neutron star with pion condensate
- quark–hybrid star
- hyperon star
- color–superconducting strange quark matter (u,d,s quarks)
- traditional neutron star
- nucleon star
- crust

F. Weber (2005)
Composition of Neutron Stars II

Lattimer and Prakash, Science (2005)
Observational Constraints

Mass-radius relationship, maximum mass

Equation of state

Cooling behavior

Phase structure, low energy degrees of freedom

Rotation

Equation of state, Viscosity

Spin-down, glitches

Superfluidity
Tolman-Oppenheimer-Volkov Equation

Structure equation in Newtonian mechanics

\[
\frac{dp}{dr} = - \frac{G \rho(r) M(r)}{r^2} = - \frac{G \epsilon(r) M(r)}{c^2 r^2}
\]

\[
M(r) = 4\pi \int_0^r r'^2 \, dr' \rho(r')
\]

Relativistic corrections

\[
\frac{dp}{dr} = - \frac{G \epsilon(r) M(r)}{c^2 r^2} \left[ 1 + \frac{p(r)}{\epsilon(r)} \right] \times \left[ 1 + \frac{4\pi r^3 p(r)}{M(r)c^2} \right] \times \left[ 1 - \frac{2GM(r)}{c^2 r} \right]^{-1}
\]

Note: All corrections are positive
Mass-Radius Relation

Lattimer and Prakash, Science (2005)
Cooling Processes

Direct URCA process

\[ n \leftrightarrow p + e^- + \bar{\nu} \] (fast)

Indirect URCA process

\[ n + n \leftrightarrow p + n + e^- + \bar{\nu} \] (slow)

Quark direct URCA process

\[ d \leftrightarrow u + e^- + \bar{\nu} \] (fast)

Collective modes

\[ \pi^\pm \rightarrow e^\pm + \nu \quad K^\pm \rightarrow e^\pm + \nu \] (very fast)

superfuidity suppresses URCA process
Cooling

Lattimer and Prakash, Science (2005)

Lattimer and Prakash, Science (2005)
Conclusion: The Many Phases of QCD

- high T
- QGP
- large N_c
- strings
- low density
- nuclear matter
- high energy
- color glas
- large density
- color superconductivity

- large isospin
- pion condensate

QCD