The “Big” Picture

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“Big” Questions

What is QCD?
What is a Phase of QCD?
What is a Plasma?
What is a (perfect) Liquid?
What is a wQGP/sQGP?
What is QCD (Quantum Chromo Dynamics)?

Elementary fields: Quarks Gluons

\[(q_\alpha)^a_f \begin{cases} 
\text{color} & a = 1, \ldots, 3 \\
\text{spin} & \alpha = 1, 2 \\
\text{flavor} & f = u, d, s, c, b, t 
\end{cases} \quad A^a_\mu \begin{cases} 
\text{color} & a = 1, \ldots, 8 \\
\text{spin} & \epsilon^\pm_\mu 
\end{cases} \]

Dynamics: Generalized Maxwell (Yang-Mills) + Dirac theory

\[\mathcal{L} = \bar{q}_f (i\slashed{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} \]

\[G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \]

\[i\slashed{D} q = \gamma^\mu \left( i\partial_\mu + g A^a_\mu t^a \right) q \]
“Seeing” Quarks and Gluons

\[ e^+ \rightarrow q \bar{q} \]

\[ e^- \rightarrow q \bar{q} \]

\[ e^+ \rightarrow q g \]

\[ e^- \rightarrow q g \]
Asymptotic Freedom

Classical field $A^\text{cl}_\mu$. Modification due to quantum fluctuations:

$$A_\mu = A^\text{cl}_\mu + \delta A_\mu$$

$$\frac{1}{g^2} F^2_{cl} \to \left( \frac{1}{g^2} + c \log \left( \frac{k^2}{\mu^2} \right) \right) F^2_{cl}$$

$$\delta A_\mu$$

$$\delta \Phi$$

**dielectric $\epsilon > 1$**

**paramagnetic $\mu > 1$**

**dielectric $\epsilon > 1$**

$$\mu \epsilon = 1 \Rightarrow \epsilon < 1$$

$$\beta(g) = \frac{\partial g}{\partial \log(\mu)} = \frac{g^3}{(4\pi)^2} \left\{ \left[ \frac{1}{3} - 4 \right] N_c + \frac{2}{3} N_f \right\} < 0$$
Running Coupling Constant

\[ \beta(g) \alpha^2 = \left( \frac{11G_F}{3} \right)^2 \frac{\alpha}{\pi} \]

\[ \alpha_s(Q) \]

\[ \alpha_s(M_Z) = 0.118 \pm 0.003 \]

\( Q/\text{[GeV]} \)
What is a Phase of QCD? Phases of Gauge Theories

\[ V(r) \sim -\frac{e^2}{r} \quad V(r) \sim -\frac{e^{-mr}}{r} \quad V(r) \sim kr \]

Standard Model: \( U(1) \times SU(2) \times SU(3) \)
What is a Phase of QCD? Phases of Gauge Theories

Coulomb
Higgs
Confinement

\[ V(r) \sim -\frac{e^2}{r} \]
\[ V(r) \sim -\frac{e^{-mr}}{r} \]
\[ V(r) \sim kr \]

QCD: High \( T \) phase
High \( \mu \) phase
Low \( T, \mu \) phase
## Phases of Matter: Symmetries

<table>
<thead>
<tr>
<th>phase</th>
<th>order param</th>
<th>broken symmetry</th>
<th>rigidity phenomenon</th>
<th>Goldstone boson</th>
</tr>
</thead>
<tbody>
<tr>
<td>crystal</td>
<td>$\rho_k$</td>
<td>translations</td>
<td>rigid</td>
<td>phonon</td>
</tr>
<tr>
<td>magnet</td>
<td>$\vec{M}$</td>
<td>rotations</td>
<td>hysteresis</td>
<td>magnon</td>
</tr>
<tr>
<td>superfluid</td>
<td>$\langle \Phi \rangle$</td>
<td>particle number</td>
<td>supercurrent</td>
<td>phonon</td>
</tr>
<tr>
<td>supercond.</td>
<td>$\langle \psi \psi \rangle$</td>
<td>gauge symmetry</td>
<td>supercurrent</td>
<td>none (Higgs)</td>
</tr>
<tr>
<td>$\chi_{sb}$</td>
<td>$\langle \bar{\psi} \psi \rangle$</td>
<td>chiral symmetry</td>
<td>axial current</td>
<td>pion</td>
</tr>
</tbody>
</table>
Chiral Symmetry

Define left and right handed fields

\[ \psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\psi \]

Fermionic lagrangian, \( M = \text{diag}(m_u, m_d, m_s) \)

\[
\mathcal{L} = \bar{\psi}_L (i\gamma_\mu \partial^\mu) \psi_L + \bar{\psi}_R (i\gamma_\mu \partial^\mu) \psi_R \\
+ \bar{\psi}_L M \psi_R + \bar{\psi}_R M \psi_L
\]

\( M = 0 \): Chiral symmetry \((L, R) \in SU(3)_L \times SU(3)_R\)

\[ \psi_L \rightarrow L\psi_L, \quad \psi_R \rightarrow R\psi_R \]
Chiral Symmetry Breaking

Chiral symmetry is spontaneously broken

\[ \langle \bar{\psi}_L^f \psi_R^g + \bar{\psi}_L^f \psi_R^g \rangle \sim -(230 \, \text{MeV})^3 \delta^{fg} \]

\[ SU(3)_L \times SU(3)_R \rightarrow SU(3)_V \quad (G \rightarrow H) \]

Consequences: dynamical mass generation \( m_Q = 300 \, \text{MeV} \gg m_q \)

\[ m_N = 890 \, \text{MeV} + 45 \, \text{MeV} \quad (\text{QCD, 95\%}) + (\text{Higgs, 5\%}) \]

Goldstone Bosons: Consider broken generator \( Q_5^a \)

\[ [H, Q_5^a] = 0 \quad Q_5^a |0\rangle = |\pi^a\rangle \quad H|\pi^a\rangle = HQ_5^a |0\rangle = Q_5^a H |0\rangle = 0 \]
Phase Diagram: Minimal Version

critical endpoint \((E)\) persists even if \(m \neq 0\)
Transitions without change of symmetry: Liquid-Gas

Phase diagram of water

Characteristics of a liquid

Pair correlation function

Good fluid: low viscosity

\[ F_x = \eta A \frac{\partial u_x}{\partial y} \]
Transitions without change of symmetry: Gas-Plasma

Phase diagram of hydrogen

Plasma Effects

Debye screening

\[ V(r) = -\frac{e}{r}e^{-m_D r} \]

\[ m_D^2 = \frac{4\pi e^2 n}{kT} \]

Plasma oscillations

\[ \omega_{pl} = \frac{4\pi e^2 n}{m} \]
Fluids: Gases, Liquids, Plasmas, . . .

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.

Historically: Water $(\rho, \epsilon, \vec{\pi})$
Example: Simple Fluid

Conservation laws: mass, energy, momentum

\[ \frac{\partial \rho}{\partial t} + \nabla (\rho \vec{v}) = 0 \]

\[ \frac{\partial \epsilon}{\partial t} + \nabla \vec{j}^\epsilon = 0 \]

\[ \frac{\partial}{\partial t} (\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0 \]

[Euler/Navier-Stokes equation]

Constitutive relations: Energy momentum tensor

\[ \Pi_{ij} = P \delta_{ij} + \rho v_i v_j + \eta \left( \partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right) + \ldots \]

reactive dissipative
Weakly Coupled Fluids: Kinetics

Weakly coupled fluid \(\equiv\) Collection of Quasi-Particles

\[ l_{\text{mf}} \gg l_{pp} \text{ and } E \gg \Gamma \]

Introduce distribution function \(f_p(x, t)\)

\[ N = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_p} f_p \]

\[ T_{ij} = \int \frac{d^3 p}{(2\pi)^3} \frac{p_ip_j}{2E_p} f_p \]
Transport from Kinetics

Boltzmann equation

\[
\frac{\partial f_p}{\partial t} + \vec{v} \cdot \nabla_x f_p + \vec{F} \cdot \nabla_p f_p = C[f_p]
\]

Collision term \( C[f_p] = C_{\text{gain}} - C_{\text{loss}} \)

Linearized theory (Chapman-Enskog): \( f_p = f_p^0 (1 + \chi_p / T) \)

suitable for transport coefficients

shear viscosity \( \chi_p = g_p p_x p_y \partial_x v_y \)
Effective Theories for Fluids (Here: Weak Coupling QCD)

\[ \mathcal{L} = \bar{q}_f (i\mathcal{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} \]

\[ \frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p = C[f_p] \quad (\omega < T) \]

\[ \frac{\partial}{\partial t} (\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0 \quad (\omega < g^4T) \]
And now for something completely different . . .

**STRING THEORY SUMMARIZED:**

I just had an awesome idea. Suppose all matter and energy is made of tiny, vibrating “strings.”

Okay. What would that imply?

I dunno.
Gauge Theory at Strong Coupling: Holographic Duals

The AdS/CFT duality relates

large \( N_c \) (Conformal) gauge theory in 4 dimensions

\[ \text{correlation fcts of gauge invariant operators} \]

\[ \exp \int dx \, \phi_0 \mathcal{O} = Z_{\text{string}}[\phi(\partial \text{AdS}) = \phi_0] \]

The correspondence is simplest at strong coupling \( g^2 N_c \)

strongly coupled gauge theory \( \Leftrightarrow \) classical string theory
Holographic Duals at Finite Temperature

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT temperature $\Leftrightarrow$ Hawking temperature of black hole

CFT entropy $\Leftrightarrow$ Hawking-Bekenstein entropy $\sim$ area of event horizon

\[
s(\lambda \to \infty) = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s(\lambda = 0)
\]

Gubser and Klebanov

\[\lambda = g^2 N\]
Holographic Duals: Transport Properties

Thermal (conformal) field theory $\equiv AdS_5$ black hole

- CFT entropy $\Leftrightarrow$ Hawking-Bekenstein entropy
  $\sim$ area of event horizon

- Shear viscosity $\Leftrightarrow$ Graviton absorption cross section
  $\sim$ area of event horizon

Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Son and Starinets

Strong coupling limit universal? Provides lower bound for all theories?
Effective Theories (Strong coupling)

\[ \mathcal{L} = \bar{\lambda}(i\sigma \cdot D)\lambda - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \ldots \iff S = \frac{1}{2\kappa_5^2} \int d^5 x \sqrt{-g} \mathcal{R} + \ldots \]

\[ \frac{\partial}{\partial t} (\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0 \quad (\omega < T) \]
Kinetics vs No-Kinetics

Spectral function \( \rho(\omega) = \text{Im}G_R(\omega, 0) \) associated with \( T_{xy} \)

- weak coupling QCD
- strong coupling AdS/CFT

transport peak vs no transport peak
Summary (Theory)

Lattice QCD: single chiral and deconfinement crossover transition

\[ T_c \sim 185 \text{ MeV}, \quad \epsilon_{cr} \sim 1.5 \text{ GeV/fm}^3 \]

Weakly coupled Quark Gluon Plasma

Quark and gluon quasi-particles, \( \gamma \ll \omega \)
Thermodynamics: Stefan-Boltzmann gas
Transport: long equilibration times, \( \eta/s \sim 1/\alpha_s^2 \gg 1 \)

Strongly coupled plasma

No quasi-particles, no kinetics, only hydrodynamics
Thermodynamics: Stefan-Boltzmann law
Transport: fast equilibration, \( \eta/s \sim 1/(4\pi) < 1 \)