Phase Structure
and Transport Properties
of (Very) Dense QCD Matter

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Schematic Phase Diagram

T

μ_e

μ

nuclear matter

neutron matter

quark matter
High Density Quark Matter

Goal: What is the densest (2nd densest, 3rd densest, ...) phase of (three flavor) quark matter?

What are the properties (thermodynamics, transport, ...) of these phases?

Are these properties consistent with observational constraints? Do they provide unique signatures?

Strategy: Weak coupling/effective field theory methods.
Very Dense Matter: Effective Field Theories

\[ p = p_F \]

\[ QCD \]

\[ p = p_F \]

\[ HDET \]

\[ HDET/NonFL \]

\[ CFLChTh \]

\[ p = p_F \]
High Density Effective Theory

Effective field theory on $v$-patches

$$\psi_{v^\pm} = e^{-i\mu v \cdot x} \left( \frac{1 \pm \vec{\alpha} \cdot \vec{v}}{2} \right) \psi$$

Effective lagrangian for $p_0 < m$

$$\mathcal{L} = \psi_v^\dagger \left( i\nu \cdot D - \frac{D^2}{2\mu} \right) \psi_v + \mathcal{L}_{4f} - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \mathcal{L}_{HDL}$$

$$\mathcal{L}_{HDL} = -\frac{m^2}{2} \sum_v G^a_{\mu\alpha} \frac{\nu^\alpha \nu^\beta}{(\nu \cdot D)^2} G^b_{\mu\beta}$$
Non-Fermi Liquid Expansion

Scale momenta \((k_0, k_{\|}, k_\perp) \rightarrow (sk_0, s^{2/3}k_{\|}, s^{1/3}k_\perp)\)

\[
[\psi] = 5/6 \quad [A_i] = 5/6 \quad [S_0] = [S_{HDL}] = 0
\]

Scaling behavior of vertices

Systematic expansion in \(\epsilon^{1/3} \equiv (\omega/m)^{1/3}\)
Vertex Corrections, Migdal’s Theorem

Corrections to quark gluon vertex

\[ \sim g v \left(1 + O(\epsilon^{1/3})\right) \]

Analogous to electron-phonon coupling

Can this fail? Yes, if external momenta fail to satisfy \( p_\perp \gg p_0 \)

\[ = \frac{e g^2}{9 \pi^2} v_\mu \log (\epsilon) \]
Superconductivity

Same phenomenon occurs in anomalous self energy

\[
\begin{align*}
\frac{g^2}{18\pi^2} \int dq_0 \log \left( \frac{\Lambda_{BCS}}{|p_0 - q_0|} \right) \frac{\Delta(q_0)}{\sqrt{q_0^2 + \Delta(q_0)^2}}
\end{align*}
\]

\[\Lambda_{BCS} = 256\pi^4 g^{-5} \mu \text{ determined by electric exchanges}\]

Have to sum all planar diagrams, non-planar suppressed by \(\epsilon^{1/3}\)

Solution at next-to-leading order (includes normal self energy)

\[\Delta_0 = 2\Lambda_{BCS} \exp \left(-\frac{\pi^2 + 4}{8}\right) \exp \left(-\frac{3\pi^2}{\sqrt{2}g}\right) \Delta_0 \sim 50 \text{ MeV}\]
Consider $N_f = 3$ ($m_i = 0$)

$$\langle q_i^a q_j^b \rangle = \phi \epsilon^{abI} \epsilon_{ijI}$$

$$\langle ud \rangle = \langle us \rangle = \langle ds \rangle$$

$$\langle rb \rangle = \langle rg \rangle = \langle bg \rangle$$

Symmetry breaking pattern:

$$SU(3)_L \times SU(3)_R \times [SU(3)]_C \times U(1) \rightarrow SU(3)_{C+F}$$

All quarks and gluons acquire a gap $[8] + [1]$ fermions, $Q$ integer
Towards the real world: Non-zero strange quark mass

Have \( m_s > m_u, m_d \): Unequal Fermi surfaces

\[ \delta p_F \simeq \frac{m_s^2}{2p_F} \]

Also: If \( p_F^s < p_F^{u,d} \) have unequal densities

Charge neutrality not automatic

Strategy

Consider \( N_f = 3 \) at \( \mu \gg \Lambda_{QCD} \) (CFL phase)

Study response to \( m_s \neq 0 \)

Constrained by chiral symmetry
Mass Terms: Match HDET to QCD

\[ \mathcal{L} = \psi_R^\dagger \frac{MM^\dagger}{2\mu} \psi_R + \psi_L^\dagger \frac{M^\dagger M}{2\mu} \psi_L \]

\[ + \frac{V_0^M}{\mu^2} (\psi_R^\dagger M \lambda^a \psi_L) (\psi_R^\dagger M \lambda^a \psi_L) \]

mass corrections to FL parameters \( \hat{\mu}_{L,R} \) and \( V^0(\overline{R}R \rightarrow \overline{L}L) \)
EFT in the CFL Phase

Consider HDET with a CFL gap term

\[ \mathcal{L} = \text{Tr} \left( \psi_L^\dagger (iv \cdot D) \psi_L \right) + \frac{\Delta}{2} \left\{ \text{Tr} \left( X^\dagger \psi_L X^\dagger \psi_L \right) - \kappa \left[ \text{Tr} \left( X^\dagger \psi_L \right) \right]^2 \right\} + (L \leftrightarrow R, X \leftrightarrow Y) \]

\[ \psi_L \rightarrow L\psi_L C^T, \quad X \rightarrow LX C^T, \quad \langle X \rangle = \langle Y \rangle = 1 \]

Quark loops generate a kinetic term for \( X, Y \)

Integrate out gluons, identify low energy fields (\( \xi = \Sigma^{1/2} \))

\[ \Sigma = XY^\dagger \]

[8]+[1] GBs

\[ N_L = \xi (\psi_L X^\dagger) \xi^\dagger \]

[8]+[1] Baryons
Effective theory: (CFL) baryon chiral perturbation theory

\[ \mathcal{L} = \frac{f_\pi^2}{4} \left\{ \text{Tr} \left( \nabla_0 \Sigma \nabla_0 \Sigma^\dagger \right) - v_\pi^2 \text{Tr} \left( \nabla_i \Sigma \nabla_i \Sigma^\dagger \right) \right\} 
+ A \left\{ [\text{Tr} (M \Sigma)]^2 - \text{Tr} (M \Sigma M \Sigma) + h.c. \right\} 
+ \text{Tr} \left( N^\dagger i v^\mu D_\mu N \right) - D \text{Tr} \left( N^\dagger v^\mu \gamma_5 \{ A_\mu, N \} \right) 
- F \text{Tr} \left( N^\dagger v^\mu \gamma_5 [ A_\mu, N ] \right) + \frac{\Delta}{2} \left\{ \text{Tr} (NN) - [\text{Tr} (N)]^2 \right\} \]

\[ \nabla_0 \Sigma = \partial_0 \Sigma + i \hat{\mu}_L \Sigma - i \Sigma \hat{\mu}_R \]
\[ D_\mu N = \partial_\mu N + i [V_\mu, N] \]

\[ f_\pi^2 = \frac{21 - 8 \log 2}{18} \frac{\mu^2}{2\pi^2} \quad v_\pi^2 = \frac{1}{3} \quad A = \frac{3\Delta^2}{4\pi^2} \quad D = F = \frac{1}{2} \]
Phase Structure and Spectrum

Phase structure determined by effective potential

\[ V(\Sigma) = \frac{f^2}{2} \text{Tr} \left( \hat{\mu}_L \Sigma \hat{\mu}_R \Sigma^\dagger \right) - A \text{Tr}(M \Sigma^\dagger) - B_1 \left[ \text{Tr}(M \Sigma^\dagger) \right]^2 + \ldots \]

\[ V(\Sigma_0) \equiv \text{min} \]

Fermion spectrum determined by

\[ \mathcal{L} = \text{Tr} \left( N^\dagger i\nu^\mu D_\mu N \right) + \text{Tr} \left( N^\dagger \gamma_5 \rho_A N \right) + \frac{\Delta}{2} \left\{ \text{Tr} \left( NN \right) - \left[ \text{Tr} \left( N \right) \right]^2 \right\}, \]

\[ \rho_{V,A} = \frac{1}{2} \left\{ \xi \hat{\mu}_L \xi^\dagger \pm \xi^\dagger \hat{\mu}_R \xi \right\} \quad \xi = \sqrt{\Sigma_0} \]
Phase Structure of CFL Phase

QCD realization of s-wave meson condensation

Driven by strangeness oversaturation of CFL state

\[ m_{s}^{\text{crit}} \approx 3.03 m_{d}^{1/3} \Delta^{2/3} \]
\[ m^{*} \approx 0.017 \alpha_{s}^{4/3} \Delta \]
Fermion Spectrum

\[ m_s^{\text{crit}} \sim (8\mu\Delta/3)^{1/2} \]

gapless fermion modes (gCFLK)

(chromomagnetic) instabilities?
Instabilities

Consider meson current

\[ \Sigma(x) = U_Y(x) \Sigma_K U_Y(x)^\dagger \quad U_Y(x) = \exp(i \phi_K(x) \lambda_8) \]

\[ \vec{\nabla}(x) = \frac{\vec{\nabla} \phi_K}{4} (-2 \hat{l}_3 + 3 \hat{Y}) \quad \vec{A}(x) = \vec{\nabla} \phi_K (e^{i \phi_K} \hat{u} + e^{-i \phi_K} \hat{u}^{-}) \]

Gradient energy

\[ \mathcal{E} = \frac{f}{2} \int \frac{v^2}{\pi} j^2_K \quad j_K = \vec{\nabla} \phi_K \]

Fermion spectrum

\[ \omega_l = \Delta + \frac{l^2}{2\Delta} - \frac{4 \mu_s}{3} - \frac{1}{4} \vec{v} \cdot j_K \]

\[ \mathcal{E} = \frac{\mu^2}{2\pi^2} \int dl \int d\hat{\Omega} \omega_l \Theta(-\omega_l) \]
Energy Functional

\[
\frac{3\mu_s - 4\Delta}{\Delta}_{\text{crit}} = a_{\text{crit}} \quad \frac{JK}{\Delta} = c_{\text{crit}}
\]

Current strongly suppressed by electric charge neutrality

\[m_s^2 \sim 2\mu\Delta: \text{multiple currents? crystalline state?}\]
Transport Properties

Dissipative Terms

\[ \dot{E} = -\frac{\eta}{2} \int d^3x \left( \partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right)^2 \]

\[ -\zeta \int d^3x (\partial_i v_i)^2 - \frac{\kappa}{T} \int d^3x (\partial_i T)^2 \]

Relevant to r-mode damping

Neutrino emissivity

\[ \epsilon_\nu = \frac{dE_\nu}{dt d^3x} \]

Relevant to cooling (together with \( \kappa, c_\nu \))
Kinetics: Quasi-particles

Quasi-particles control transport of flavor, energy, and momentum.

Neutrino emissivity, neutrino mean free path:

Shear viscosity, thermal conductivity
Bulk viscosity: Weak interaction

Bulk viscosity: Strong interaction
Umpaired Quark Matter

\[ \kappa \simeq 0.5 \frac{m_D^2}{\alpha_s^2} \]

\[ \eta \simeq 4.4 \times 10^{-3} \frac{\mu^4 m_D^{2/3}}{\alpha_s^2 T^{5/3}} \]

\[ \zeta \simeq \frac{\alpha T^2}{\omega^2 + \beta T^4} \quad (\alpha, \beta^{1/2} \sim G_F^2) \]

\[ \epsilon_\nu \simeq \frac{457}{630} \alpha_s G_F^2 T^6 \mu_e \mu_u \mu_d \]

Non-FLT

Non-FLT

QURCA

FLT, Non-FLT
CFL Quark Matter

\[ \eta = 1.3 \times 10^{-4} \frac{\mu^8}{T^5} \]

phonons

\[ \zeta = 0.011 \frac{M_s^4}{T} \]

phonons

\[ \zeta = \frac{C \gamma_K}{\omega^2 + \gamma_K^2} \quad (\gamma_K \sim G_F^2 f_K^2) \]

weak kaon decay

\[ \epsilon_\pi \sim A G_F^2 f_\pi^2 m_\pi n_\pi \]

pion (kaon) decay

phonons, kaons

\[ \kappa \sim ? \]
Summary

Systematic weak coupling expansion for $\Delta/m_D, T/m_D \ll 1$.
(Non-Fermi Liquid Regime)

$$\Delta \gg \delta \mu: \text{Color-flavor-locked (CFL) phase}$$

Regime $\delta \mu < \Delta$ controlled by chiral symmetry, but the regime $\delta \mu \sim \Delta$ is complicated

s and p-wave meson condensation

Issues not covered in this talk: Transition to nuclear matter, nuclear exotics, etc.

Constraints from compact star phenomenology