Perfect Fluidity in Cold Atomic Gases?

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Elliptic Flow

Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy.
Requires “perfect” fluidity ($\eta/s < 0.1$)?

(s)QGP saturates (conjectured) universal bound $\eta/s = 1/(4\pi)$?
Viscosity Bound: Rough Argument

Kinetic theory estimate of shear viscosity

\[
\eta \sim \frac{1}{3} n \bar{u} m l = \frac{2}{3} n \left( \frac{1}{2} m \bar{u}^2 \right) \frac{l}{\bar{u}} = \frac{2}{3} n \epsilon \tau_{mft}
\]

Entropy density: \( s \sim k_B n \). Uncertainty relation implies

\[
\frac{\eta}{s} \sim \frac{\epsilon \tau_{mft}}{k_B n} \sim \frac{E \tau_{mft}}{k_B} \geq \frac{\hbar}{k_B}
\]

Validity of kinetic theory as \( E \tau \sim \hbar \)?

Why \( \eta/s \)? Why not \( \eta/n \)?
Holographic Duals at Finite Temperature

Thermal (conformal) field theory \( \equiv \text{AdS}_5 \) black hole

- CFT temperature \( \Leftrightarrow \)
- CFT entropy \( \Leftrightarrow \)

Hawking temperature of black hole

Hawking-Bekenstein entropy \( \sim \) area of event horizon

Strong coupling limit

\[
s = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s_0
\]

Gubser and Klebanov

Extended to transport properties by Policastro, Son and Starinets
Quark Gluon Plasma Equation of State (Lattice)

Compilation by F. Karsch (SciDAC)
Holographic Duals: Transport Properties

Thermal (conformal) field theory $\equiv$ $AdS_5$ black hole

CFT entropy $\Leftrightarrow$ Hawking-Bekenstein entropy

$\sim$ area of event horizon

shear viscosity $\Leftrightarrow$ Graviton absorption cross section

$\sim$ area of event horizon

Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Son and Starinets

Strong coupling limit universal? Provides lower bound for all theories?
Viscosity Bound: Common Fluids
Viscosity Bound: Counter Examples?

non relativistic systems: can make $S/N$ large

\[
\frac{\eta}{s} = \frac{1}{\log(N_s)} \frac{c\sqrt{mT}}{a^2 n}
\]

modified conjecture: applies to systems that can be embedded in a relativistic (gauge?) theory

T. Cohen: Consider heavy-light mesons in QCD with $N_F = N_c \to \infty$

\[
m_Q = m_Q^0 N_F \quad n = \frac{n_0}{\log(N_F)} \quad T = \frac{T_0}{N_F \log(N_F)^{1/2}}
\]

\[
\frac{\eta}{s} \sim \frac{1}{\log(N_s)} \quad \text{stable fluid?}
\]
Designer Fluids

Atomic gas with two spin states: "↑" and "↓"

Feshbach resonance

\[ a(B) = a_0 \left( 1 + \frac{\Delta}{B - B_0} \right) \]

"Unitarity" limit \( a \to \infty \)

\[ \sigma = \frac{4\pi}{k^2} \]
Why are these systems interesting?

System is intrinsically quantum mechanical

cross section saturates unitarity bound

System is scale invariant at unitarity. Universal thermodynamics

\[ \frac{E}{A} = \xi \left( \frac{E}{A} \right)_0 = \xi \frac{3}{5} \left( \frac{k_F^2}{2M} \right) \]

System is strongly coupled but dilute

\[
(k_F a) \rightarrow \infty \quad (k_F r) \rightarrow 0
\]

Strong elliptic flow observed experimentally
Elliptic Flow

Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy.
Collective Modes

Radial breathing mode

Ideal fluid hydrodynamics, equation of state $P \sim n^{5/3}$

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) = 0$$
$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} = -\frac{1}{mn} \vec{\nabla} P - \frac{1}{m} \vec{\nabla} V$$

$$\omega = \sqrt{\frac{10}{3}} \omega_{\perp}$$
Damping of Collective Excitations

\[ T/T_F = (0.5, 0.33, 0.17) \]

\[ \tau \omega: \text{ decay time } \times \text{ trap frequency} \]

Kinast et al. (2005)
Viscous Hydrodynamics

Energy dissipation ($\eta, \zeta, \kappa$: shear, bulk viscosity, heat conductivity)

$$\dot{E} = -\frac{\eta}{2} \int d^3x \left( \partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right)^2$$

$$- \zeta \int d^3x (\partial_i v_i)^2 - \frac{\kappa}{2} \int d^3x (\partial_i T)^2$$

Shear viscosity to entropy ratio

(assuming $\zeta = \kappa = 0$)

$$\frac{\eta}{s} = \frac{3}{4} \xi^{\frac{1}{2}} (3N)^{\frac{1}{3}} \frac{\Gamma}{\omega_{\perp}} \frac{\bar{\omega}}{\omega_{\perp}} \frac{N}{S}$$

see also Bruun, Smith, Gelman et al.
Damping dominated by shear viscosity?

Study dependence on flow pattern

Study particle number scaling

viscous hydro: $\Gamma \sim N^{-1/3}$  
Boltzmann: $\Gamma \sim N^{1/3}$

Role of thermal conductivity?

suppressed for scaling flows: $\delta T \sim T(\delta n/n) \sim \text{const}$
Elliptic Flow

Free scaling expansion

\[ n(r_\perp, r_z) = \frac{1}{b_\perp^2 b_z} n_0 \left( \frac{r_\perp}{b_\perp}, \frac{r_z}{b_z} \right) \]

\[ \dot{b}_\perp = \frac{\omega_\perp^2}{b_\perp (b_\perp^2 b_z)^\gamma} \]

Viscous damping

\[ \dot{E} = -\frac{4}{3} \left( \frac{\dot{b}_\perp}{b_\perp} - \frac{\dot{b}_z}{b_z} \right)^2 \int d^3 x \eta(x) \]

\[ \Delta E = \int dt \dot{E} \] converges quickly
Elliptic Flow (cont)

Can define $v_2 = \langle \cos(2\phi) \rangle$ as in HI collisions

$$\epsilon = \frac{\langle 2z^2 - x^2 + y^2 \rangle}{\langle z^2 + x^2 + y^2 \rangle}$$

Can also sweep to BEC regime and simulate recombination models
Final Thoughts

Cold atomic gases provide interesting, strongly coupled, model system in which to study sources of dissipation.

\[ \frac{\eta}{s} \sim 1/3 \]

Smaller than any other known liquid (except for QGP?). Since other sources of dissipation exist, this is really an upper bound.

There are reliable calculations of \( \frac{\eta}{s} \) at high \( T \) (Bruun, Smith, \ldots) and low \( T \) (Rupak and T.S, in prep). Extrapolate to \( T \sim T_F \)
Conjectured bound has a smooth non-relativistic limit. Note that the $a \to \infty$ limit can also be realized in QCD (by tuning $\mu, \mu_e$ and $m_q$).

But: In non-relativistic systems $s \gg n$ possible

Purely field theoretic proofs?

$\mathcal{N} = 4$ SUSY YM is special because there is no phase transition. In real systems there is a phase transition as the coupling becomes large, and the new phase (confined in QCD, superfluid in the atomic system) has weakly coupled low energy excitations, and a large viscosity.

No quasi-particles in sQGP?