Instantons and the Spin-Flavor Structure of Hadrons

Thomas Schaefer

North Carolina State
Quantum Chromodynamics

Elementary fields:

<table>
<thead>
<tr>
<th>Quarks</th>
<th>Gluons</th>
</tr>
</thead>
<tbody>
<tr>
<td>(q_α)^a_f</td>
<td>A^a_μ</td>
</tr>
<tr>
<td>color \ a = r, b, g</td>
<td>color \ a = 1, \ldots, 8</td>
</tr>
<tr>
<td>spin \ \alpha = \uparrow, \downarrow</td>
<td>spin \ \epsilon^\pm_μ</td>
</tr>
<tr>
<td>flavor \ f = u, d, s, c, b, t</td>
<td></td>
</tr>
</tbody>
</table>

Dynamics: Dirac + generalized Maxwell theory (Yang-Mills theory)

\[
\mathcal{L} = \bar{q}_f (i \slashed{D} - m_f) q_f - \frac{1}{4} G^{a}_{\mu \nu} G^{a}_{\mu \nu}
\]

\[
G^{a}_{\mu \nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc}_{\ \ \ } A^b_\mu A^c_\nu
\]

\[
i \slashed{D} q = \gamma^\mu (i \partial_\mu + g A^a_\mu t^a) q
\]
“Seeing” Quarks and Gluons
Asymptotic Freedom

Classical field \( A_0^{cl} \sim g/r \). Modification due to quantum fluctuations:

\[
A_\mu = A_\mu^{cl} + \delta A_\mu
\]

\[
g \rightarrow g(\mu)
\]

\[
\beta(g) = \frac{\partial g}{\partial \log(\mu)}
\]

\[
dielectric \ \epsilon > 1 \quad \text{paramagnetic} \ \mu > 1 \quad \text{dielectric} \ \epsilon > 1
\]

\[
\mu \epsilon = 1 \implies \epsilon < 1
\]

\[
\beta(g) = \frac{g^3}{(4\pi)^2} \left\{ \left[ \frac{1}{3} - 4 \right] N_c + \frac{2}{3} N_f \right\}
\]
Running Coupling Constant

\[ \beta(g) \alpha_s - (11G_F/3) \frac{\alpha_s}{\pi} \]

\[ \alpha_s(Q) = 0.118 \pm 0.003 \]
What is a proton?

Why does this picture “work”? Large $N_c$ limit (?)

Where does it fail? Why? OZI violation, flavor mixing
The Structure of the Proton

The mass of the Proton (from DIS, trace anomaly)

\[ E_q = \langle p | \int d^3 x (-i \vec{ \alpha } \cdot \vec{D}) | p \rangle \approx 310 \text{ MeV} \]

\[ E_g = \langle p | \int d^3 x \frac{1}{2} (E^2 + B^2) | p \rangle + \ldots \approx 545 \text{ MeV} \]

\[ E_m = \langle p | \int d^3 x (m_u \bar{u}u + m_d \bar{d}d) | p \rangle \approx 45 \text{ MeV} \]

Gluon field strength is large

\[ \langle p | E^2 | p \rangle \approx 1700 \text{ MeV} \quad \langle p | B^2 | p \rangle \approx -1050 \text{ MeV} \]

and approximately self-dual
number of quark-anti-quark pairs is large

\[ \langle p | \bar{u}u + \bar{d}d | p \rangle = \frac{\Sigma_{\pi N}}{m} \simeq 6 \]

... and not flavor symmetric

\[ \frac{\bar{d}(x)}{\bar{u}(x)} \simeq 2 \quad \text{(NuSea, ...)} \]

quark contribution to proton spin is small

\[ \Delta \Sigma = \Delta u + \Delta d + \Delta s = (0.25 \pm 0.1) \quad \text{(SMC, SLAC, Hermes)} \]

\[ \langle p | \bar{q} \gamma_\mu \gamma_5 q | p \rangle = \Delta q s_\mu \]

... and strange quarks are polarized \( \Delta s = -0.12 \)
Hadronic Correlation Functions

hadronic current \( j_M(x) = \bar{q}(x) \Gamma q(x) \)

\[ \Pi(x) = \langle j(x)j(0) \rangle \]

short distance behavior: OPE

\[ \Pi(Q) = c_0 \log(Q^2) + c_4 \frac{\langle \mathcal{O}_4 \rangle}{Q^4} + c_6 \frac{\langle \mathcal{O}_6 \rangle}{Q^6} + \ldots \]

experimental information

\[ \Pi(Q) = \int ds \frac{\rho(s)}{s + Q^2} \]
Vector Channels: $\rho$ and $a_1$

\[
1 + \frac{\alpha}{\pi} \left( -\langle G G \rangle - \langle q \bar{q}^2 \rangle \right) \cdot x^4
\]

\[
1 - \frac{\alpha}{\pi} \left( -\langle G G \rangle + \langle q \bar{q}^2 \rangle \right) \cdot x^4
\]

\[
\frac{\Pi(x)}{\Pi(0)}
\]

\[
\rho
\]

\[
a_1
\]
Scalar Channels: $\pi$ and $\delta$

$$\Pi(x)/\Pi^0(x) = 1 + \frac{\alpha}{\pi} + (\#<GG> + \#s\bar{q}q^2) \cdot x^4$$

$\pi$ lattice
$\delta$ lattice
$\pi$ OPE
$\delta$ OPE

$\pm (L \leftrightarrow R)$
OZI violation: $\eta' - \pi, \sigma - \delta, \omega - \rho, a_1 - f_1$
Summary

Only small effects in \((\bar{L}L \pm \bar{R}R)^2\).

Sign changes for \(\bar{L}R \leftrightarrow \bar{R}L\).

Sign changes for \((\bar{u}d)(\bar{u}d) \leftrightarrow (\bar{u}u)(\bar{d}d)\).

\[
\mathcal{L} = G \det_f(\bar{\psi}_L \psi_R) + (L \leftrightarrow R)
\]
Topological in QCD

classical potential is periodic in variable $X$

\[ X = \int d^3x K_0(x, t) \]

\[ \partial^\mu K_\mu = \frac{1}{32\pi^2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu} \]
classical minima correspond to pure gauge configurations

\[ A_i(x) = iU^\dagger(x)\partial_i U(x) \]
\[ E^2 = B^2 = 0 \]

semi-classical tunneling paths: Instantons

\[ A^a_\mu(x) = 2\frac{\eta_{a\mu\nu}x_\nu}{x^2 + \rho^2}, \]
\[ G^a_{\mu\nu} \tilde{G}^a_{\mu\nu} = \frac{192\rho^4}{(x^2 + \rho^2)^4}. \]
(Anti)Instantons: Dirac operator has a L/R zero mode.

\[ \gamma \cdot (\partial + A_{I,A}) \psi^0_{L,R} = 0 \]

axial charge violation:
\[ \Delta Q_A = 2 \]
instanton induced quark interaction ($N_f = 2$)

\[ \mathcal{L} = G \det_f (\bar{\psi}_L, f \psi_R, g) \]

\[ G = \int d\rho n(\rho) \]

violates $U(1)_A$ but preserves $SU(2)_{L,R}$

\[ \text{... and contributes to the } \eta' \text{ mass} \]

tunneling rate (barrier penetration factor)

\[ n(\rho) \sim \exp \left[ -\frac{8\pi^2}{g^2(\rho)} \right] \sim \rho^{b-5} \]
Instanton Ensemble

instanton liquid described by partition function

\[ Z = \frac{1}{N_I! N_A!} \prod_{I} \int [d\Omega_I \, n(\rho_I)] \times \det(\mathcal{D}) \exp(-S_{int}) \]

quark propagator

\[ S(x, y) = \sum_{IJ} \psi_I(x) \left( \frac{1}{T + im} \right)_{IJ} \psi_J^\dagger(y) + S_{NZM}(x, y) \]
Meson Correlation Functions

\[ m_\pi = 140^* \text{ MeV} \quad (f_\pi = 71 \text{ MeV}) \]
\[ m_\rho = 795 \text{ MeV} \]
\[ m_{a_0} \simeq 1 \text{ GeV} \]
\[ m_\rho \simeq m_\omega \]
\[ m_\sigma \simeq 580 \text{ MeV} \]
\[ m_{\eta'} \simeq 1 \text{ GeV} \]
V–A Correlation Functions

Aleph spectral function
\[ \tau \rightarrow (V, A, I = 1) \nu_\tau \]

coordinate space correlator
OPE, instanton liquid, data
Large $N_c$: From extraordinary to ordinary hadrons

$\eta'$ becomes light, light $\sigma$ disappears

(quenching artifacts in $a_0$ disappear)
Quark Contribution to Nucleon Spin

polarized DIS implies large OZI violation

related to axial anomaly and instantons?

\[ g_A^0 = \Delta u + \Delta d + \Delta s \]
\[ \simeq 0.25 \]
\[ g_A^8 = \Delta u + \Delta d - 2\Delta s \]
\[ \simeq 0.65 \]

\[ \partial^\mu A_\mu^0 = \frac{N_f g^2}{16\pi^2} G^{a}_{\mu\nu} \tilde{G}^{a}_{\mu\nu} \]

\[ g_A^0 = \frac{N_f}{32\pi^2 m_N} \langle p| g^2 G^{a}_{\mu\nu} \tilde{G}^{a}_{\mu\nu} |p\rangle \]
OZI violation

Suppression of $g_A^0$ property of the nucleon or of the QCD vacuum?

\[
(g_Q^0 - (g_Q^3)^3) 
\]

Study singlet correlators in $\bar{q}q$ and $\bar{Q}q$ (or $QQq$) channel
Vacuum Properties

Axial charge screening related to topological charge screening?

\[ \chi_{top} = \frac{1}{V} \langle Q_{top}^2 \rangle = 0 \]

\[ L \rightarrow R(\bar{L}R) \]

e.g. Veneziano and Shore

\[ g_A^0 = g_A^8 \sqrt{\frac{6\chi'_{top}(0)}{f_\pi^2}} \] (target independent)

also: Shuryak and Forte, Dorokhov and Kochelev
Numerical Study

\[ g_A^3 \approx 1.25 \] agrees with experiment
\[ g_A^0 \approx 0.75 \] too large (little OZI violation)
(\bar{q}q) and (\bar{Q}q) states

\begin{align*}
(f^2 m^2)^0 &< (f^2 m^2)^3 \\
(g_A^Q)^0 &> (g_A^Q)^3
\end{align*}

Note:  
1. \( f_1/a_1 \) and \( g_A^0/g_A^3 \) anti-correlated (\( \rightarrow \) NJL studies)  
2. sign fixed by QCD inequalities
Summary and Outlook

instantons account for OZI violation in meson sector

Not all hadrons are alike

instanton liquid reproduces axial vector coupling $g_A$

But: $g_A^8 \approx g_A^0 \approx 0.75$

no evidence that suppression of $g_A^0$ is a vacuum effect

$$[(g_A^Q)^0 \sim 1] > [(g_A^Q)^3 \sim 0.9]$$

OZI violation? Go back and look at $g_A^8$

Large $SU(3)_F$ violation?