Formulas and Numerical Constants

Lorentz transformation: The system $S'$ is moving with velocity $(v_x, v_y, v_z) = (v, 0, 0)$ relative to the $S$ system. The Lorentz transformations are

$$
x' = \gamma (x - vt), \quad y' = y, \quad z' = z, \\
t' = \gamma \left( t - \frac{v}{c^2} x \right),
$$

where $\gamma = 1/(1 - \beta^2)^{1/2}$ and $\beta = v/c$. The inverse Lorentz transformation corresponds to $v \rightarrow -v$.

Velocity addition: An object moves with velocity $(u_x, u_y, u_z)$ in the $S$-system. The components of the velocity in the $S'$-system are

$$
u'_x = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}, \quad u'_y = u_y \gamma \left( 1 - \frac{vu_x}{c^2} \right), \quad u'_z = u_z \gamma \left( 1 - \frac{vu_x}{c^2} \right).
$$

The inverse transformation corresponds to $v \rightarrow -v$.

Relativistic kinematics: In the following $m$ always refers to the rest mass of a particle

$$
E^2 = p^2 c^2 + m^2 c^4, \\
E = \gamma mc^2 \quad p = \gamma mv,
$$

and $\gamma = (1 - v^2/c^2)^{-1}$. The four vector $(E, \vec{p}c)$ transforms under Lorentz transformations like the four vector $(ct, \vec{x})$:

$$
p'_x = \gamma \left( p_x - vE/c^2 \right), \quad p'_y = p_y, \quad p'_z = p_z, \\
E' = \gamma (E - vp_x),
$$

Black Body Radiation: Planck’s law is

$$
R(\lambda) = \frac{2\pi \hbar c^2}{\lambda^5} \frac{1}{e^{\frac{\hbar c}{k_B T\lambda}} - 1}.
$$
The maximum of the distribution occurs at $\lambda T = 2.898 \cdot 10^{-3}$ K m. The total emissivity is given by the Stefan-Boltzmann law

$$R = \sigma T^4, \quad \sigma = \frac{\pi^2 k_B^4}{60h^3c^2}$$  \hfill (10)

De Broglie relations: De Broglie postulated the following relations between $(E, p)$ and $(\lambda, f)$

$$E = hf, \quad (E = \hbar \omega)$$  \hfill (11)
$$p = h/\lambda, \quad (p = \hbar k)$$  \hfill (12)

where $\hbar = h/(2\pi)$.

Bohr’s model: Bohr’s model of hydrogen like atoms is based on the quantization condition $L = mvr = n\hbar$. The allowed energies and radii are

$$r_n = \frac{n^2 a_0}{Z}, \quad a_0 = \frac{\hbar^2}{m_e k c^2},$$  \hfill (13)
$$E_n = -\frac{Z^2 E_0}{n^2}, \quad E_0 = \frac{m_e k^2 e^4}{2\hbar^2},$$  \hfill (14)

where $k = 1/(4\pi \varepsilon_0)$ is the constant of proportionality in Coulomb’s law, $e$ is the charge, and $m_e$ is the mass of the electron. $Z$ is the charge of the nucleus (in units of $e$). The constant $a_0$ is called the Bohr radius. The quantity

$$\alpha = \frac{ke^2}{\hbar c} \simeq \frac{1}{137}$$  \hfill (15)

is called the fine structure constant.

Schrödinger equation: The time-dependent and time-independent Schrödinger equations are

$$i\hbar \frac{\partial}{\partial t} \psi(x, t) = \left\{ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right\} \psi(x, t),$$  \hfill (16)
$$E\psi(x) = \left\{ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right\} \psi(x).$$  \hfill (17)
The wave function is related to the probability
\[
P(x, t) \, dx = \psi^*(x, t)\psi(x, t) \, dx.
\] (18)

More generally, expectation values are given by
\[
\langle O \rangle = \int dx \, \psi^*(x, t) \hat{O} \psi(x, t).
\] (19)

An important example is the momentum \(p \)
\[
\langle p \rangle = \int dx \, \psi^*(x, t) \left( -i\hbar \frac{\partial}{\partial x} \right) \psi(x, t).
\] (20)

3d Schrödinger equation: Solutions of the Schrödinger equation for a potential with rotational symmetry have the form
\[
\psi(r, \theta, \phi) = R_{nl}(r) Y_{lm}(\theta, \phi),
\] (21)
where \( Y_{lm} \) are the spherical harmonics, \((l, m)\) label \( L^2 = \hbar^2 (l + 1) \) and \( L_z = \hbar m \) \((m \leq l)\), and \( R_{nl}(r) \) is the radial wave function (labeled by the quantum number \( n \)). The ground state wave function of the hydrogen atom is
\[
R_{10}(r) = \frac{2}{\sqrt{a_0^3}} e^{-r/a_0}, \quad Y_{00}(\theta, \phi) = \frac{1}{\sqrt{4\pi}},
\] (22)
where \( a_0 \) is the Bohr radius defined above.

Numerical Constants:
\[
\begin{align*}
k_B &= 1.381 \times 10^{-23} \text{ J/K} = 8.617 \times 10^{-5} \text{ eV/K} \\
\hbar &= 6.626 \times 10^{-34} \text{ J} \cdot \text{s} \\
c &= 2.998 \times 10^8 \text{ m/sec} \\
hc &= 1240 \text{ eV} \cdot \text{nm} \\
hc &= 197.33 \text{ MeV} \cdot \text{fm} \\
e &= 1.602 \times 10^{-19} \text{ C} \\
1 \text{ cal} &= 4.186 \text{ J} \\
1 \text{ eV} &= 1.602 \times 10^{-19} \text{ J} \\
1 \text{ u} &= 1.661 \times 10^{-27} \text{ kg} = 931.49 \text{ MeV/c}^2 \\
m_e c^2 &= 512 \text{ keV} \\
m_p c^2 &= 935 \text{ MeV}
\end{align*}
\] (23)