PY 203 Sample Final Exam

Short Answers (3 points each):

1. What is the gauge boson mediating the decay shown in this Feynman diagram?

\[ \nu_e \rightarrow \mu^- \]

2. To prevent a satellite from becoming electrically charged, it is coated with platinum, a metal with a large work function of 5.32 eV. What will be the longest wavelength of sunlight that can eject an electron from the surface of the satellite?

\[ \frac{hc}{\lambda} = 5.32 \text{ eV} \quad \lambda = \frac{1240 \text{ eV} \cdot \text{nm}}{5.32 \text{ eV}} = 233 \text{ nm} \]

3. What would be the kinetic energy of an electron with a deBroglie wavelength of 5nm?

\[ \lambda = \frac{h}{P} = \frac{h}{\frac{2\pi}{\lambda}} = \frac{1240 \text{ eV} \cdot \text{nm}}{5 \text{ nm}} = 248 \text{ eV} \quad E = \frac{p^2}{2m} = \frac{(248 \text{ eV})^2}{2 \times 911,000 \text{ eV}} \approx 0.06 \text{ eV} \]

4. Draw arrows connecting each quark combination to the hadron it represents.

\[ \begin{array}{cccc}
\text{ud} & \rightarrow & \pi^+ \\
\text{udd} & \rightarrow & \pi^0 \\
\text{ud} & \rightarrow & \pi^- \\
\text{qd} & \rightarrow & p
\end{array} \]

5. What is the longest wavelength in the Paschen Series (end state of n=3)?

\[ \Delta E = 13.6 \text{ eV} \left( \frac{1}{3^2} - \frac{1}{2^2} \right) = 0.661 \text{ eV} \quad \lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} \cdot \text{nm}}{0.661 \text{ eV}} = 1876 \text{ nm} \]

6. Calculate the binding energy per nucleon for Helium \(^{3}\text{Li}\) (atomic mass 3.0160293u).

\[ \Delta mc^2 = (3m_p + 4m_n - 7.016003u)c^2 = 37.8 \text{ MeV} \quad \text{divide by 7 nucleons} \Rightarrow \frac{5.39 \text{ MeV}}{\text{nucleon}} \]

7. A sample of \(^{144}\text{Nd}\) has a mass of 0.05394 kg and emits an average of 2.36 \(\alpha\) particles per second. Determine the mean lifetime of \(^{144}\text{Nd}\).

\[ \frac{N}{N_0} e^{-t/\tau} \quad \frac{t}{\tau} = \ln \left( \frac{N}{N_0} \right) = \ln \left( \frac{4.6 \times 10^{15}}{8.14 \times 10^{15}} \right) = 1.73 \]

\[ \tau = 20 \text{ days} = 11,550 \text{ days} \]

8. Sketch the two lowest energy wavefunctions for the potential shown below.

- First excited state
- Ground state

\[ t_y = (\ln 2) \tau \]

\[ = 8 \text{ days} \]
Quantitative Questions (16 points each):

9. A particle of mass 1.00 MeV/c^2 and kinetic energy 3.00 MeV collides with a stationary particle of mass 3.00 MeV/c^2. After the collision, the particles stick together.
   a. Find the momentum (in units of MeV/c) and energy (in units of MeV) of each particle in this system before the collision.
   b. What is the total momentum and energy of the system after the collision?
   c. What is the rest mass energy of the composite particle formed in the collision?
   d. What is the kinetic energy of the composite particle after the collision?

\[ E = 4 \text{ MeV} \quad 3 \text{ MeV} \]
\[ p c = \sqrt{E^2 - (mc^2)^2} \]
\[ = \sqrt{16mc^2 - (mc^2)^2} \]
\[ = \sqrt{16mc^2 - (mc^2)^2} = \sqrt{15} mc = 7.5 \text{ MeV} \]

After collision, total energy and momentum are unchanged

\[ E = 7 \text{ MeV} \]
\[ p = 7.5 \text{ MeV/c} \]

Find rest mass of composite particle from

\[ (mc^2)^2 = E^2 - p^2 c^2 = 49 \text{ MeV}^2 - 15 \text{ MeV}^2 = 34 \text{ MeV}^2 \]

\[ m = \sqrt{34 \text{ MeV/c}^2} \]

Kinetic energy of composite particle is \( E_{\text{total}} - mc^2 \)

\[ KE = 7 \text{ MeV} - 1.17 \text{ MeV} = 1.17 \text{ MeV} \]
10. The $K^0$ particle has a mass of 497.7 MeV/c$^2$. It decays into a $\pi^-$ and a $\pi^+$, each having mass 139.6 MeV/c$^2$. Following the decay of the $K^0$ particle, one of the pions is at rest in the laboratory frame.

a. What is the kinetic energy of the other pion after the decay?

b. What is the kinetic energy of the $K^0$ particle prior to decay?

\[
m_{K^0} c^2 = 2 \gamma m_{\pi^-} c^2 \Rightarrow \gamma_{\pi^-} = m_{K^0} \rightarrow m_{\pi^-} = 1.78\]

\[
in \text{CM frame pion has } E = \gamma_{\pi^-} m_{\pi^-} c^2, \quad \mathbf{p} = \gamma m_{\pi^-} \mathbf{v}_{\pi^-}.
\]

Transform to rest frame of other pion

\[
E' = \gamma_{\pi^+} \left( \gamma_{\pi^-} m_{\pi^-} c^2 + \gamma_{\pi^-} m_{\pi^+} c^2 \right) = \gamma_{\pi^+} m_{\pi^-} c^2 \left(1 + \beta_{\pi^-}^2\right)
\]

\[
= \gamma (139.6 \text{ MeV})(2- \frac{1}{2} \beta_{\pi^-}^2) = 745.3 \text{ MeV}
\]

Kinetic energy = $E' - m_{\pi^-} c^2 = 605.4 \text{ MeV}$

\[
\text{Kinetic energy of } K^0 \quad E - m_{K^0} c^2 = 0.78 (497.7 \text{ MeV}) = 383 \text{ MeV}
\]

11. Consider a proton moving in a one-dimensional potential well with $V(x) = 0.5 kx^2$. The ground state wave function is $\psi(x) = A_0 \exp(-ax^2)$.

a. Verify that $\psi(x)$ is a solution to Schrödinger's equation.

b. What is the ground state energy?

c. Compute the normalization constant $A_0$ for the ground state.

d. Compute the expectation value $\langle x^2 \rangle$ for the ground state.

\[
\psi = A_0 e^{-ax^2}
\]

\[
\psi'' = -2a \psi' + 4a^2 \psi = -\frac{h^2}{2m} \psi + \frac{1}{2} k x^2 \psi = E \psi
\]

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\]

\[
\psi(x) = A_0 e^{-ax^2} \quad (\frac{-2a \pi^2}{m} + \frac{1}{2} k) x^2 = (E - \frac{h^2}{m})
\]

\[
\text{Normalization: } 1 = \int_{-\infty}^{\infty} \psi^* \psi \, dx = A_0^2 \int_{-\infty}^{\infty} e^{-2ax^2} \, dx = A_0^2 \frac{\sqrt{\pi}}{2a} \quad A_0 = \frac{\sqrt{2}a}{\pi} \quad A_0 = \left(\frac{2a}{\pi}\right)^{1/4}
\]

\[
\langle x^2 \rangle = \int_{-\infty}^{\infty} \psi^* x^2 \psi \, dx = \frac{1}{\sqrt{2a}} \int_{0}^{\infty} x^2 e^{-x^2} \, dx = \frac{1}{\sqrt{2a}} \int_{0}^{\infty} u^2 e^{-u^2} \, du = \frac{1}{\sqrt{2a}} \left[ \frac{u^2}{2} e^{-u^2} \right]_{0}^{\infty} = \frac{1}{4a} \langle x^2 \rangle
\]
12. A blackbody at temperature $T_1$ radiates energy at a power level of 2 mW. The same object, when at a temperature of $2T_1$, radiates at a power level of
a. 1 mW  
 b. 2 mW  
 c. 4 mW  
 d. 16 mW  
 e. 32 mW

13. A lump of clay whose rest mass is 4 kg is traveling at three-fifths the speed of light when it collides head-on with an identical lump going the opposite direction with the same speed. If the two lumps stick together and no energy is radiated away, what is the mass of the composite clump?
2 x 1.25 x 4
a. 4 kg  
 b. 6.4 kg  
 c. 8 kg  
 d. 10 kg  
 e. 13.3 kg

14. If the total energy of a particle of mass $m$ is equal to three times its rest energy, then the magnitude of the particle’s relativistic momentum is
a. $mc / 3$  
 b. $mc / 2$  
 c. $2mc$  
 d. $8\sqrt{2}mc$  
 e. $3mc$

15. A free particle with initial kinetic energy $E$ and de Broglie wavelength $\lambda$ enters a region in which there is potential energy $V$. What is the particle’s new de Broglie wavelength?
$\lambda^2 = \frac{h^2}{2mE}$
$\lambda = \frac{h}{\sqrt{2mE}}$

16. Be transforms into Li by
a. emitting an alpha particle only  
 b. emitting an electron only  
 c. emitting a proton only  
 d. emitting a neutron only  
 e. electron capture by the nucleus with emission of a neutrino

17. The mean kinetic energy of the conduction electrons in metals is ordinarily much higher than $kT$ because
a. electrons have more degrees of freedom than atoms  
 b. the electrons and the lattice are not in thermal equilibrium  
 c. the electrons form a degenerate Fermi gas  
 d. electrons in metals are highly relativistic  
 e. electrons interact via phonons

18. The muon decays with a characteristic lifetime of about $10^6$ seconds into an electron, a muon neutrino, and an electron antineutrino. The muon is forbidden from decaying into an electron and just a single neutrino by the law of conservation of
a. charge  
 b. mass  
 c. energy and momentum  
 d. baryon number  
 e. lepton number

19. The solution to the Schrödinger equation for the ground state of hydrogen is
$\psi_0 = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$
In spherical symmetry the probability distribution is given by $4\pi r^2 |\psi|^2$. What is the most probable value for $r$?
$\frac{1}{4\pi} \int r^2 |\psi|^2 dV = 0$
a. 0  
 b. $a_0/2$  
 c. $a_0$  
 d. $2a_0$  
 e. $\infty$

20. A beam of electrons is accelerated through a potential difference of 25 kilovolts in an x-ray tube. The continuous x-ray spectrum will have a short-wavelength limit of most nearly
$\lambda = \frac{h}{E}$
$\lambda = \frac{h}{2mE}$
$25 \text{ keV} \approx 31 \text{ keV}$
$E = \frac{50}{2m}$