10-24) Stars larger than the Sun can expire in spectacular explosions called supernovae, leaving behind a collapsed remnant of the star called a neutron star. These neutron stars have masses comparable to the original masses of the stars but radii of only a few kilometers! The high rotation rates (1 rev/s to 1000 rev/s) are assumed to be due to the conservation of angular momentum during the collapse. (a) Using data from the textbook, estimate the rotation rate of the Sun if it were to collapse into a neutron star of radius 10 km. The Sun is not a uniform sphere of gas, and its moment of inertia is given by $I = 0.059 MR^2$. Assume that the neutron star is spherical and has a uniform mass distribution. (b) Is the rotational kinetic energy of the Sun greater or smaller after the collapse? By what factor does it change, and where does the energy go to or come from? [330 rev/s]
10-52) Two disks of identical mass but different radii \((r\text{ and } 2r)\) are spinning on frictionless bearings at the same angular speed \(\omega_0\), but in opposite directions. The two disks are brought slowly together. The resulting frictional force between the surfaces eventually brings them to a common angular velocity. 

(a) What is the magnitude of that final angular velocity in terms of \(\omega_0\)?

(b) What is the change in rotational kinetic energy of the system? Explain. [\(\omega_f = 0.6\omega_0, \Delta K = -0.64K_i\)]

---

There are no external forces or torques, so the net angular momentum of the system remains constant.

Initial angular momentum \(\vec{L} = I_1\omega_0 - I_2\omega_0\) (disk 2 is in opposite direction)

After the come into contact, both disks move with the same rotation rate, \(\omega_f\)

\[\vec{L} = (I_1 + I_2)\omega_f\]

Setting final = initial angular momentum gives \(\omega_f = \omega_0 \frac{I_1 - I_2}{I_1 + I_2}\)

\(I\) for a uniform disk is \(\frac{1}{2}mr^2\):

\[I_1 = \frac{1}{2}m(2r)^2 = 2mr^2\]

\[I_2 = \frac{1}{2}m(r)^2 = \frac{1}{2}mr^2\]

\[\omega_f = \omega_0 \frac{2mr^2 - \frac{1}{2}mr^2}{2mr^2 + \frac{1}{2}mr^2} = \omega_0 \frac{3}{5}\]

\[= \frac{3}{5}\omega_0\]

Change in kinetic energy \(\Delta K = K_f - K_i\)

\[= \frac{1}{2}(I_1 + I_2)\omega_f^2 - \left(\frac{1}{2}I_1\omega_0^2 + \frac{1}{2}I_2\omega_0^2\right)\]

Substitute \(\omega_f\) = \(\frac{3}{5}\omega_0\)

\[\Delta K = \frac{1}{2}(I_1 + I_2)\left(\frac{3}{5}\omega_0\right)^2 - \left(\frac{1}{2}I_1\omega_0^2 + \frac{1}{2}I_2\omega_0^2\right)\]

\[= \frac{9}{25}\left(I_1 + I_2\right)\omega_0^2 - \frac{1}{2}I_1\omega_0^2 - \frac{1}{2}I_2\omega_0^2\]

\[= \frac{9}{25}\left(I_1 - I_2\right)\omega_0^2\]

\[= \frac{1}{25}K_i\]

Energy lost to friction
A uniform rod is resting on a frictionless table when it is suddenly struck at one end by a sharp horizontal blow in a direction perpendicular to the rod. The mass of the rod is $M$ and the magnitude of the impulse applied by the blow is $I$.

Immediately after the rod is struck, (a) what is the velocity of the center of mass of the rod, (b) what is the velocity of the end that is struck, (c) and what is the velocity of the other end of the rod? (d) Is there a point on the rod that remains motionless?

\[
\begin{align*}
& (a) \quad v_{cm} = \frac{I}{M}, \\
& (b) \quad v = \frac{4I}{M}, \\
& (c) \quad v = -\frac{2I}{M}
\end{align*}
\]

The impact sends the rod across the table, spinning about its center of mass, which itself is moving in a straight line.

**Linear motion:** conservation of momentum says the rod is moving with a linear momentum of $P = I$. This must equal $M_{tot} \cdot V_{cm}$, so the center of mass is moving with $V_{cm} = \frac{I}{M}$.

The impact also gives the rod angular momentum $L = \vec{r} \times \vec{p}$, where $r = \frac{l}{2}$ (\(l\) is length of rod).

Since $\vec{F}$ and $\Delta \vec{p}$ are \(\perp\), $L = \frac{1}{2} \Delta \vec{p} = \frac{1}{2} \vec{L} \cdot \vec{I}$.

Using $L = I \omega$, where $I = \frac{1}{12} M l^2$ for a rod (Table 9.1),

\[
\omega = \frac{I}{I} = \frac{\frac{1}{2} M l}{\frac{1}{12} M l^2} = 6 \frac{I}{M l}
\]

the velocity of each end of the rod is $v = \omega r = \omega \left(\frac{l}{2}\right)$, but in opposite directions, and relative to the center of mass.

\[
\begin{align*}
V_{top} &= V_{cm} + \omega \frac{l}{2} = \frac{I}{M} + 6 \frac{I}{M l} \frac{l}{2} = 4 \frac{I}{M} \\
V_{bot} &= V_{cm} - \omega \frac{l}{2} = \frac{I}{M} - 3 \frac{I}{M} = -2 \frac{I}{M}
\end{align*}
\]

**d)** No, every point is moving after impact.
10-66) A projectile of mass $M_p$ is traveling at a constant velocity $v_0$ toward a stationary disk of mass $M$ and radius $R$ that is free to rotate its axis. Before impact, the projectile is traveling along a line displaced a distance $b$ below the axis. The projectile strikes the disk and sticks to point $B$. Model the projectile as a point mass. 

(a) Before impact, what is the total angular momentum of the disk–projectile system about the axis? Answer the following questions in terms of the symbols given at the start of this problem. 

(b) What is the angular speed $\omega$ of the disk–projectile system just after the impact? 

(c) What is the kinetic energy of the disk–projectile system after impact? 

(d) How much mechanical energy is lost in this collision?

Before impact the disk is not moving, so the total angular momentum is only that of the projectile:

for a point mass $L = \vec{r} \times \vec{p} = \vec{b} \times \vec{m}_p \vec{v}_0$ coming out of the page

After impact the system has the same angular momentum, $b m_p v_0$, but now $L = I \omega$ where $I = I_{disk} + I_{proj}$

$$I = \frac{1}{2} m R^2 + m_p R^2$$

$$\omega = \frac{L}{I} = \frac{b v_0 m_p}{R^2 (\frac{1}{2} m + m_p)} = \frac{b v_0}{R^2 (1 + m m_p)}$$

$$K_{after} = \frac{1}{2} I \omega^2 = \frac{L^2}{2 I} = \frac{b^2 v_0^2 m_p^2}{m R^2 + 2 m_p R^2} = \frac{1}{2} m_p v_0^2 \left( \frac{b^2}{R^2} \right) \left( \frac{1}{1 + m m_p} \right)$$

$$\Delta K = K_{after} - K_{init} \quad \text{where} \quad K_{init} = \frac{1}{2} m_p v_0^2$$

so energy lost is

$$\frac{1}{2} m_p v_0^2 \left( 1 - \frac{b^2/R^2}{1 + m m_p} \right)$$
11-52) An object is projected straight upward from the surface of Earth with an initial speed of 4.0 km/s. What is the maximum height it reaches? [940 km]

Using conservation of energy,

\[ E_{\text{ini}} = E_{\text{fin}} \]

where \( U = -\frac{GMm}{r} \)

and \( r = R_\oplus + \text{height} \)

\[ H = ? \]

\[ R_\oplus = 6378 \text{ km} \]

\[
\begin{align*}
\text{initial} & \quad K & \quad U \\
\frac{1}{2}mv_0^2 & \quad -\frac{GMm}{R_\oplus} \\
\text{final} & \quad \phi & \quad -\frac{GMm}{R_\oplus + H}
\end{align*}
\]

\[
\begin{align*}
\frac{1}{2}mv_0^2 + -\frac{GMm}{R_\oplus} &= -\frac{GMm}{R_\oplus + H} \\
\frac{1}{2}v_0^2 &= \frac{-GM}{R} = \frac{-GM}{R + H} \\
\frac{v_0^2}{2GM} - \frac{1}{R} &= \frac{-1}{R + H} \\
R + H &= \frac{1}{\frac{v_0^2}{2GM} + \frac{1}{R}}
\end{align*}
\]

\[
H = \frac{1}{\frac{v_0^2}{2GM} + \frac{1}{R}} - R_\oplus
\]

\[
H = \left( \frac{(4 \times 10^3 \text{ m/s})^2}{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(6 \times 10^24 \text{ kg})} + \frac{1}{6.378 \times 10^6 \text{ m}} \right)^{-1} - 6.378 \times 10^6 \text{ m}
\]

\[ H = 7.31 \times 10^6 - 6.378 \times 10^6 = 932 \text{ km} \]
11.59) Many satellites orbit Earth at maximum altitudes above Earth’s surface of 1000 km or less. Geosynchronous satellites, however, orbit at an altitude of 35790 km above Earth’s surface. How much more energy is required to launch a 500-kg satellite into a geosynchronous orbit than into an orbit 1000 km above the surface of Earth? [11.1 GJ]

\[ R_E = 6378 \text{ km} \]
\[ h = 1000 \text{ km} \]
\[ d = 35790 \text{ km} \]

Energy of an object in orbit:
\[ \frac{1}{2} U = -\frac{GMm}{2r} \]

So energy of geosynchronous orbit compared to low-earth orbit is:
\[ \frac{1}{2} U_{\text{geo}} - \frac{1}{2} U_{\text{leo}} = -\frac{1}{2} \frac{GM_\odot M_s}{R_\odot + 35790 \text{ km}} + \frac{1}{2} \frac{GM_\odot M_s}{R_\odot + 1000 \text{ km}} \]

\[ \Delta E = \frac{GM_\odot M_s}{2R_\odot} \left( \frac{1}{1 + \frac{1000 \text{ km}}{R_\odot}} - \frac{1}{1 + \frac{35790 \text{ km}}{R_\odot}} \right) \]

\[ = \left( \frac{6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}}{2 \times (6.378 \times 10^6 \text{ m})^2} \right) \left[ \frac{1}{1 + \frac{1000}{6378}} - \frac{1}{1 + \frac{35790}{6378}} \right] \]

\[ = 1.57 \times 10^{10} \left[ \frac{1}{1.157} - \frac{1}{6.611} \right] = 1.11 \times 10^{10} \text{ J} \]

\[ \Delta E = 11.2 \text{ GJ} \]