PHY 125, useful formulas

1. velocity and acceleration:

\[ \ddot{v} = \frac{d\vec{r}}{dt}, \quad \ddot{a} = \frac{d\ddot{v}}{dt} \]  \hspace{1cm} (1)

linear motion with constant acceleration

\[ x = x_0 + v_0 t + \frac{1}{2} a t^2 \]  \hspace{1cm} (2)
\[ v = v_0 + a t \]  \hspace{1cm} (3)

also: \( v^2 = v_0^2 + 2a(x - x_0) \)

2. Newton’s laws

\[ \sum_i \vec{F}_i = 0 \Rightarrow \ddot{v} = \text{const} \]  \hspace{1cm} (4)
\[ \sum_i \vec{F}_i = m \ddot{a} \]  \hspace{1cm} (5)
\[ \vec{F}_{12} = -\vec{F}_{21} \]  \hspace{1cm} (6)

3. static friction (\( N \) normal force)

\[ F_s \leq \mu_s N \]  \hspace{1cm} (7)

kinetic friction

\[ F_k = \mu_k N \]  \hspace{1cm} (8)

4. Newton’s law of gravity

\[ \vec{F} = -\frac{GmM}{r^2} \hat{r} \]  \hspace{1cm} (9)
5. centripetal acceleration

\[ a_c = \frac{v^2}{r} \]  \hspace{1cm} (10)

6. Work

\[ W = \int \vec{F} \cdot d\vec{r} \]  \hspace{1cm} (11)

constant force \( W = \vec{F} \cdot d \vec{r} \)

7. kinetic energy \( K = \frac{1}{2}mv^2 \). Energy conservation for conservative forces

\[ K_1 + U_1 = K_2 + U_2, \]  \hspace{1cm} (12)

where \( U = - \int \vec{F} \cdot d\vec{r} + \text{const} \) is the potential energy associated with the conservative force.

8. Hooke’s law (spring constant \( k \))

\[ F_s = -kx \]  \hspace{1cm} (13)

elastic potential energy \( E = \frac{1}{2}kx^2 \).

9. Gravitational potential energy

\[ U(r) = -\frac{GmM}{r} \]  \hspace{1cm} (14)

Approximate result near the surface of the earth: \( U = mgh \).

10. Power \( P = \frac{\Delta W}{\Delta t} \).

11. Linear momentum \( \vec{p} = m\vec{v} \). Newton’s equation of motion

\[ \sum_i \vec{F}_i = \frac{d\vec{p}}{dt} \]  \hspace{1cm} (15)

Momentum Conservation: \( \vec{P} = \sum_i \vec{p}_i \)

\[ \frac{d\vec{P}}{dt} = \sum_i \vec{F}^\text{ext}_i. \]  \hspace{1cm} (16)

Then: \( \vec{F}^\text{ext} = 0 \Rightarrow \vec{P} = \text{const.} \)
12. rotation around a fixed axis

\[ \omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt}. \]  

(17) constant angular acceleration

\[ \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2. \]  

(18)

13. A particle in a rigid body rotating with angular velocity \( \omega \) has tangential velocity given by

\[ v = r\omega \]  

(19)

The tangential and radial acceleration are

\[ a_{\text{tan}} = r\alpha, \quad a_{\text{rad}} = \frac{v^2}{r} = \omega^2 r. \]  

(20)

14. The moment of inertia of a rigid body about a given axis is defined by

\[ I = \sum_i m_i r_i^2. \]  

(21)

The angular momentum is

\[ L = I \omega \]  

(22)

and the rotational kinetic energy

\[ K = \frac{1}{2} I \omega^2. \]  

(23)

15. When a force \( \vec{F} \) acts on a body, the torque of that force with respect to a point \( O \) is given by

\[ \tau = Fl \]  

(24)

where \( l \) is the lever arm. A definition involving vectors is \( \vec{\tau} = \vec{r} \times \vec{F}. \) The angular acceleration is related to the torque by

\[ \tau = I \alpha. \]  

(25)
16. The angular momentum of a point particle is given by
\[ \vec{L} = \vec{r} \times \vec{p} \]  
(26)

In relation between torque and the change of angular momentum is
\[ \vec{\tau} = \frac{d\vec{L}}{dt} . \]  
(27)

17. Useful numbers
\[ g = 9.81 \frac{m}{s^2} \]  
(28)
\[ G = 6.67 \cdot 10^{-11} \frac{Nm^2}{s^2} \]  
(29)
\[ M_E = 5.97 \cdot 10^{24} kg \]  
(30)
\[ r_E = 6.38 \cdot 10^6 m. \]  
(31)

Simple math

1. Circle of radius \( r \)
\[ A(\text{rea}) = \pi r^2, \quad C(\text{circumference}) = 2\pi r \]  
(32)

2. two-dimensional vector \( \vec{A} = A_x \vec{i} + A_y \vec{j} \)
\[ A = |\vec{A}| = \sqrt{A_x^2 + A_y^2} \]  
(33)
\[ A_x = A \cos(\theta) \]  
(34)
\[ A_y = A \sin(\theta) \]  
(35)
\[ \tan(\theta) = \frac{A_y}{A_x} \]  
(36)

where \( \theta \) is the angle between \( \vec{A} \) and \( \vec{i} \).
3. scalar product: $\vec{A} = A_x\hat{i} + A_y\hat{j}$, $\vec{B} = B_x\hat{i} + B_y\hat{j}$

$$\vec{A} \cdot \vec{B} = AB \cos(\theta) = A_x B_x + A_y B_y.$$  \hfill (37)

4. quadratic equation $ax^2 + bx + c = 0$ has solutions

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$  \hfill (38)

5. If $y = ax^n$ then

$$\frac{dy}{dx} = anx^{n-1}.$$  \hfill (39)