Homework 4, due 10-6

1. A particle of mass $m$ moves in the potential

$$V(x) = \begin{cases} 
V_1 & x < 0 \\
0 & 0 < x < a \\
V_2 & a < x 
\end{cases},$$

where $V_1, V_2 > 0$.

(a) Find the equation that determines the energy levels.

(b) Consider the case $V_1 = V_2$. Draw a graph that shows the eigenvalue equation.

(c) Is there always at least one bound state? How does the number of bound states scale with $V$?

(d) Derive an approximate analytical expression for the energy level in a shallow potential, that means in the case that the potential $V = V_1 = V_2$ allows just one bound state with energy $E \simeq V$. Compute the first correction to to $E \simeq V$.

2. A particle of mass $m$ moves in the potential $V = -V_0 \delta(x)$ ($V_0 > 0$).

(a) Find the energy and wave function of the groundstate.

(b) Are there any excited bound states?

Hint for part a: First consider the Schrödinger equation in the regime $x > 0$ and $x < 0$. Then determine the boundary condition at $x = 0$. 