Homework 8, due 11-4

1. Consider a particle governed by the Hamiltonian

\[ H = \frac{p^2}{2m} + V(x). \]

Consider \([H, x]\) in order to show that

\[ \sum_m (E_m - E_n)|\langle n | x | m \rangle|^2 = \frac{\hbar^2}{2m}, \]

where \(|n\rangle\) are energy eigenstates satisfying \(H|n\rangle = |n\rangle E_n\). This relation is known as the dipole (or Thomas-Reiche-Kuhn) sum rule. Check the dipole sum rule in the case of the 1-dimensional harmonic oscillator.

2. Consider the 1-dimensional harmonic oscillator with Hamiltonian \(H = \frac{p^2}{2m} + \frac{m}{2} \omega^2 x^2\). Show that the classical action is given by

\[ S_{cl} = \frac{m \omega}{2 \sin(\omega(t_b - t_a))} \left\{ \left( x_a^2 + x_b^2 \right) \cos(\omega(t_b - t_a)) - 2x_a x_b \right\}, \]

where \(x_{cl}(t_a) = x_a\) and \(x_{cl}(t_b) = x_b\). What is the meaning of the singularity when \(\omega(t_b - t_a) = n\pi\)?

3. In class we defined the infinitesimal translation operator through its action on coordinate eigenstates, \(T(\epsilon)|x\rangle = |x + \epsilon\rangle\). Compute

\[ T^\dagger(\epsilon) X T(\epsilon), \quad T^\dagger(\epsilon) P T(\epsilon), \]

where \(X, P\) are the coordinate and momentum operators.

4. Consider the Hamiltonian

\[ H = \frac{p^2}{2m} + \frac{1}{2a^2} m \omega^2 (x^2 - a^2)^2. \]

Construct approximate wave functions for the ground state and the first excited state in the limit \(a \to \infty\). Explain your reasoning using parity symmetry.