Homework 5, due 10-4

In class we introduced product wave functions

\[ \uparrow \uparrow = \chi_{\uparrow}(1)\chi_{\uparrow}(2) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \]

Spin operators act on product spin wave functions as follows

\[ \sigma_{x,1}\sigma_{y,2}\chi_{\uparrow}(1)\chi_{\uparrow}(2) = [\sigma_{x}\chi_{\uparrow}(1)] [\sigma_{y}\chi_{\uparrow}(2)]. \]

Expectation values are defined as

\[ (\chi_{\uparrow}(1)\chi_{\uparrow}(2))^\dagger \sigma_{x,1}\sigma_{y,2}\chi_{\uparrow}(1)\chi_{\uparrow}(2) = \left[\chi_{\uparrow}^\dagger(1)\sigma_{x}\chi_{\uparrow}(1)\right] \left[\chi_{\uparrow}^\dagger(2)\sigma_{y}\chi_{\uparrow}(2)\right]. \]

In class we argued that \(\chi_{A,S}\)

\[ \chi_{A,S} = \frac{1}{\sqrt{2}} (\uparrow \downarrow \mp \downarrow \uparrow) \]

have spin zero and one, respectively.

1. Check this statement explicitly by computing

\[ \vec{S}^2 \chi_{A,S} \]

where

\[ \vec{S} = \vec{S}_1 + \vec{S}_2 = \frac{\hbar}{2} (\vec{\sigma}_1 + \vec{\sigma}_2). \]

2. Use your result to compute the expectation value of \(\vec{S}_1 \cdot \vec{S}_2\) in the spin zero and one states,

\[ \chi_{A}^\dagger (\vec{S}_1 \cdot \vec{S}_2) \chi_{A} = ? \]

\[ \chi_{S}^\dagger (\vec{S}_1 \cdot \vec{S}_2) \chi_{S} = ? \]

3. Suppose the potential between two nucleon is of the form

\[ V(r) = V_0(r) + (\vec{S}_1 \cdot \vec{S}_2)V_1(r). \]

What can you say about the sign and relative size \(V_0\) and \(V_1\)?