Homework 9, due 11-12

1. (The method of conformal mappings) Consider a complex function $w(z) = \Phi(x + iy) + i\chi(x + iy)$.

(a) Show that if $w(z)$ is an analytic function ($w(x, y)$ satisfies the Cauchy-Riemann differential equations) then both $\Phi$ and $\chi$ satisfy the two-dimensional Laplace equation.

(b) Show: If $\Phi$ is interpreted as the electrostatic potential then lines of constant $\chi$ represents the lines of force. Also show that the lines $\Phi = \text{const}$ and $\chi = \text{const}$ are orthogonal.

(c) Consider $w(z) = -2\log(z)$. What is the physical situation represented by $\Phi(x, y) = \text{Re} w(z)$?

(d) Find a complex function $w(z)$ that represents two parallel line charges $\pm \lambda$ in three dimensions. Calculate the surface charge density induced by a line charge located at $x = a, y = z = 0$ on a conducting plane at $x = 0$.

2. In an anisotropic medium we have $D_i = \epsilon_{ij}E_j$ where $\epsilon_{ij} = \epsilon_{ji}$ is the dielectric tensor. Determine the field of a point charge at the origin in an anisotropic medium. (Note: $\epsilon_{ij}$ can be diagonalized. This implies that it is sufficient to consider the case $\epsilon_{ij} = \text{diag}(\epsilon_x, \epsilon_y, \epsilon_z)$.)